A NOTE ON THE USE OF FUZZY LOGIC IN THE EVALUATION OF STUDENTS' TEST RESULTS

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ABSTRACT: *In this article, we describe the use of fuzzy logic and fuzzy sets in the evaluation of students' results in distance testing at the Faculty of Mathematics and Informatics of the University of Plovdiv. We are trying to reevaluate borderline grades, i.e. test scores that vary marginally but because of which students with a dissimilarity of a single point or so are assigned different grades. For this purpose, we consider an additional criterion. We compare the results obtained by the use two kinds of fuzzy functions.*

KEYWORDS: *fuzzy set, fuzzy logic, fuzzy function, test evaluation*

1 Introduction

One of the most important indicators of the quality of education is the result from it presented as a grade, based on some assessment of a learner's knowledge and skills. Grades can have various functions, for example diagnostic – for quality control; didactic – for control and evaluation; motivational – to help student get motivated for higher achievements; formative – to form self-control skills in the student, etc. However, students with similar abilities may be assigned different grades if their achievements are borderline cases, for example, a student who has scored 29 points out of 60 will fail, while another one with 30 or 31 points will pass the test, which can be viewed as unfair. In an attempt to find a reasonable and impartial solution to this problem, we have used fuzzy logic in [3, 4] to review the test results in English of 78 first-year students of Informatics. The test comprises 60 closed questions awarding one point for a correct answer, and one open question with a maximum of 4 points.

2 Fuzzy sets

The idea of fuzzy sets, introduced by Lofti Zadeh [5], is quite simple. It represents some uncertainty, which is due to imprecision or vagueness rather than to randomness. A typical example can be set $A = \{$if someone is younger than or equal to 25 years of age, we consider him/her young$\}$; set $B = \{$if someone is older than 25 years of age, we consider him/her to be old$\}$. We can define the characteristic functions of sets A and B in the following way:

$$\mu_A(x) = \begin{cases} 1, & x \leq 25 \\ 0, & x > 0 \end{cases}, \quad \mu_B(x) = \begin{cases} 0, & x \leq 25 \\ 1, & x > 0. \end{cases}$$

If a person is 16 years of age, we obtain that $\mu_A(16) = 1$ and $\mu_B(16) = 0$. Therefore, he/she belongs to the group of young people.

A problem arises if a person is 26 years of age – is he/she young or old? Now, we can modify the membership functions μ_A and μ_B to suit our example. Fuzzy sets are considered with respect to a nonempty base set X of elements of interest (in our case X will be the set of all people). The essential idea is that each element $x \in X$ is assigned a membership grade μ_A to set A, taking values in $[0, 1]$, with $\mu_A(x) = 0$ corresponding to non-membership, $0 < \mu_A(x) < 1$ to partial membership, and $\mu_A(x) = 1$ to full membership. One possible definition that provides an example of fuzzy sets of the young and old people can be:

$$\mu_A(x) = \begin{cases} 1, & x \le 20 \\ -\dfrac{x}{10}+3, & 20 < x \le 30 \\ 0, & x > 30 \end{cases}, \quad \mu_B(x) = \begin{cases} 0, & x \le 20 \\ \dfrac{x}{10}-2, & 20 < x \le 30 \\ 1, & x > 30. \end{cases}$$

We can plot the function μ_A with red color and μ_B with blue color (Figure 1).

Now, a person who is 24 years old can be viewed as partly young and partly old. In fact, we get that he/she belongs to the set of young people with a degree $\mu_A(24) = 0.6$ and to the set of old people $\mu_B(24) = 0.4$. If we consider a person of 17 years of age, he/she belongs to the set of young people with a degree $\mu_A(17) = 1$ and to the set of old people with a degree $\mu_B(17) = 0$ so he/she is definitely a young person.

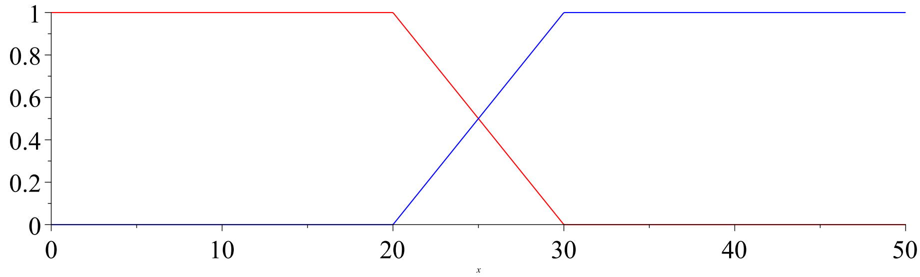

Figure 1: Plots of μ_{young} and μ_{old}.

There are different functions that can be used to assign a membership grade. The previous example illustrates the so called trapezium membership function. Owing to the use of the bell–shaped function (Figure 2), when the age of a person gets closer and closer to the limit of 30 years, the degree of his/her membership grade to the set of young people approaches 1 more rapidly, and this is evident from $\mu_{old}(27) = 0.9$ (Figure 1) and $\nu_{old}(27) = 0.97$ (Figure 2). If a person's age is close to the boundary value of 30, his/her membership grade, calculated with the help of the bell-shaped function, is closer to one than if it is calculated with a trapezium membership function. A sample for such a membership function is displayed in Figure 2.

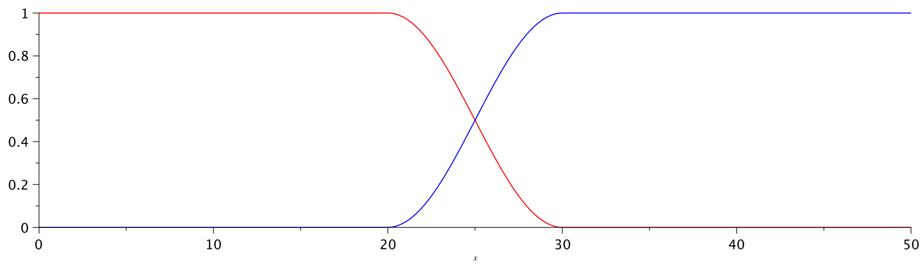

Figure 2: Plots of ν_{young} and ν_{old}.

3 Fuzzification of score metrics

As pointed out by the authors [1, 2], it is difficult and unfair to assign a Good grade to a student who has obtained, for example, 60 points, and to assign a Very good grade to another one with 61 points. Therefore, in an attempt to evaluate students' test results more fairly, the use of fuzzy sets is justifiable [3, 4]. Two different fuzzy functions were used for this purpose: trapezium functions in [3] (Figure 1) and bell–shaped functions in [4]. We will try to evaluate if there is a statistical difference in the use of the two different types of functions investigated in [3, 4].

We have considered the test results in English of 78 first-year students of Informatics. The test comprises 60 closed questions awarding one point for a correct answer, and one open question with a maximum of 4 points. The maximum total number of points is 64. By the classical marking system, a student will fail on the exam for points in the range of $[0,31]$; the range for a satisfactory grade is $[32,39]$, a good grade the range is $[40,47]$, $[48,55]$ for a very good grade, and for an excellent mark - $[56,64]$.

Let $f:\mathbb{R}^3\to\mathbb{R}$ and $g:\mathbb{R}^3\to\mathbb{R}$ be two functions.

We have defined five functions that represent the fuzzy membership functions to the sets of grades.

$$\mu_F=\mu_{Fail}=\begin{cases}1 & x<x_1\\ f(x_1,x_2,x) & x_1\le x<x_2\\ 0 & x_2\le x\le x_9\end{cases},\quad \mu_S=\mu_{Satis.}=\begin{cases}0 & x_0\le x<x_1\\ g(x_1,x_2,x) & x_1\le x<x_2\\ 1 & x_2\le x<x_3\\ f(x_3,x_4,x) & x_3\le x<x_4\\ 0 & x_4\le x\le x_9\end{cases},$$

$$\mu_G=\mu_{Good}=\begin{cases}0 & x_0\le x<x_3\\ g(x_3,x_4,x) & x_3\le x<x_4\\ 1 & x_4\le x<x_5\\ f(x_5,x_6,x) & x_5\le x<x_6\\ 0 & x_6\le x\le x_9\end{cases},\quad \mu_V=\mu_{VeryGood}=\begin{cases}0 & x_0\le x<x_5\\ g(x_5,x_6,x) & x_5\le x<x_6\\ 1 & x_6\le x<x_7\\ f(x_7,x_8,x) & x_7\le x<x_8\\ 0 & x_8\le x\le x_9\end{cases}$$

$$\mu_E=\mu_{Excellent}=\begin{cases}0 & x_0\le x<x_7\\ g(x_7,x_8,x) & x_7\le x<x_8\\ 0 & x_8\le x\le x_9\end{cases}$$

We can consider for the trapezium functions $f(a,b,x)=\frac{x-b}{a-b}$ and $g(a,b,x)=\frac{a-x}{a-b}$, and for the bell–shaped functions $f(a,b,x)=\frac{\cos\left(\frac{a\pi}{a-b}-\frac{\pi}{a-b}\right)}{2}+\frac{1}{2}$, $g(a,b,x)=\frac{\cos\left(\frac{(2a-b)\pi}{a-b}-\frac{\pi}{a-b}\right)}{2}+\frac{1}{2}$.

We need to select another criterion that depends on a student's score in order to decide what grade to assign him/her in case the points that they have received in a test do not belong definitely to a given set [3, 4]. Consequently, the second assessment that will be applied to estimate a student's knowledge will be their result of the open question.

For the fuzzy functions defined on the basis of the overall score, we consider $x_0=0$, $x_1=29.4$, $x_2=34.6$, $x_3=37.4$, $x_4=42.6$, $x_5=45.4$, $x_6=50.6$, $x_7=53.4$, $x_8=58.6$, $x_9=64$, and for the results of the open question $y_0=0$, $y_1=1.3$, $y_2=1.9$, $y_3=1.9$, $y_4=2.5$, $y_5=2.5$, $y_6=3.1$, $y_7=3.1$, $y_8=3.7$, $y_9=4$ for both the trapezium functions and the bell–shaped functions.

The Fuzzy Associative matrix [1] provides a convenient way to directly combine the input relations in order to obtain the fuzzified output results. The input values for the results from the open question are across the top of the matrix and the input values for the total score of the test are down left in the matrix, where μ (the left column) presents the fuzzy functions defined on the overall score and ν (the first row) presents the fuzzy functions defined on the open question. We have used the classical Bulgarian grading scale.

Table 1: The fuzzy associative matrix

	ν_{Fail}	$\nu_{Satisfactory}$	ν_{Good}	$\nu_{VeryGood}$	$\nu_{Excellent}$
μ_{Fail}	2	2	3	3	4
$\mu_{Satisfactory}$	2	3	3	4	4
μ_{Good}	2	3	4	5	5
$\mu_{VeryGood}$	3	4	5	5	6
$\mu_{Excellent}$	3	4	5	6	6

The rules for performing set operations of union (AND) and intersection (OR) are of most interest [1]. For union, we look at the degree of membership for each set and pick the lower one of the two, that is: $\mu_{A\cap B} = \min(\mu_A, \mu_B)$ and for intersection, we look at the degree of membership for each set and pick the higher one of the two, that is $\mu_{A\cup B} = \max(\mu_A, \mu_B)$.

In accordance with [1], we need to calculate the grade for each student, whose test is not unquestionably a part of a given set. To do so, we can refer to the table where the function F registers the minimums of μ and ν. The maximum membership grade, which is acquired from the table, represents the corrected grade from the associative matrix.

Table 2: The various combinations of the minimums of μ and ν, calculated by use of F

$F(\mu_F(p), \nu_F(q)$	$F(\mu_F(p), \nu_S(q)$	$F(\mu_F(p), \nu_G(q)$	$F(\mu_F(p), \nu_V(q)$	$F(\mu_F(p), \nu_E(q)$
$F(\mu_S(p), \nu_F(q)$	$F(\mu_S(p), \nu_S(q)$	$F(\mu_S F(p), \nu_G(q)$	$F(\mu_S(p), \nu_V(q)$	$F(\mu_S(p), \nu_E(q)$
$F(\mu_G(p), \nu_F(q)$	$F(\mu_G(p), \nu_S(q)$	$F(\mu_G(p), \nu_G(q)$	$F(\mu_G(p), \nu_V(q)$	$F(\mu_G(p), \nu_E(q)$
$F(\mu_V(p), \nu_F(q)$	$F(\mu_V(p), \nu_S(q)$	$F(\mu_V(p), \nu_G(q)$	$F(\mu_V(p), \nu_V(q)$	$F(\mu_V(p), \nu_E(q)$
$F(\mu_E(p), \nu_F(q)$	$F(\mu_E(p), \nu_S(q)$	$F(\mu_E(p), \nu_G(q)$	$F(\mu_E(p), \nu_V(q)$	$F(\mu_E(p), \nu_E(q)$

It is easy to observe that Table 2 has no more than four places where the numbers are different from zero. Let us now consider a student with an overall score of $p = 47$ points and a score of the open question of $q = 3.6$ points. Normally, he/she will get a grade Good 4. The Fuzzified grade with a trapezium (Table 3) and bell–shaped (Table 4) of a student with $p = 47$ and $q = 3.6$ are presented.

Table 3: Fuzzified with trapezium functions

0	0	0	0	0
0	0	0	0	0
0	0	0	0.07	0.51
0	0	0	0.07	0.49
0	0	0	0	0

Table 4: Fuzzified grade bell-shaped functions

0	0	0	0	0
0	0	0	0	0
0	0	0	0.005	0.485
0	0	0	0.005	0.494
0	0	0	0	0

Therefore, if the grade of the student is fuzzified with the trapezium function, he/she will get a Good (4) mark, but if the bell–shaped function is used, then the grade will be Very Good (5).

More examples are presented in [3, 4].

As commented in [3, 4], fuzzification should be used so that the distribution of the grades before and after it do not differ statistically. It is called a *fair correction* of the marks, because it guarantees that the overall score results of the students remain unchanged. After the process of fuzzification, we get that 46 marks have been fuzzified.

We check the T–test with Paired Samples before the fuzzification and after it.

The T–test with Paired Samples using bell–shaped function returns the following:

- Standard T-Test with Paired Samples
- Null Hypothesis:
- Sample drawn from populations with difference of means equal to 0
- Alt. Hypothesis:
- Sample drawn from population with difference of means not equal to 0
- Sample Size: 46
- **Result: [Accepted]**

On the other hand, the T–test with Paired Samples using bell–shaped function returns:

- Standard T-Test with Paired Samples
- Null Hypothesis:
- Sample drawn from populations with difference of means equal to 0
- Alt. Hypothesis:
- Sample drawn from population with difference of means not equal to 0
- Sample Size: 46
- **Result: [Rejected]**

4 Conclusion

We would like to point out that when the fuzzy sets get bigger, it is better to use the bell–shaped functions as long as the distributions do not differ statistically.

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