STABILITY FOR SOLITARY WAVES*

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ABSTRACT: *Solitary waves (or solitons or coherent structures) are ubiquitous objects that appear in most physically relevant time evolution models. More precisely, they are solutions of partial differential equations, depending in a special way on the time variable. There has been a lot of activity in the last fifty years regarding their existence, functional properties (such as smoothness, rate of decay etc.) and most importantly spectral/linear/nonlinear stability as solutions to the general time dependent models. In this talk, I will first review some general principles of the stability theory, and then I will discuss some recent results of mine and collaborators. .*

KEYWORDS: *Solitary waves, stability, Dispersive models*

Many equations in classical physics, mechanics and engineering are derived through their Hamiltonian formulation - namely they minimize an appropriate action functional. As such they enjoy special structure, which makes their theory especially rich, with specific properties and important dynamical consequences.

We consider, by way of introducing the important ideas, the classical non-linear Schrödinger equation (NLS),

$$iu_t + \Delta u + F(|u|^2)u = 0, x \in \mathbb{R}^n, t > 0 \tag{1}$$

and the (generalized) Korteweg-deVries equation (KdV)

$$v_t + v_{xxx} + F(|v|^2)v = 0. \tag{2}$$

for some appropriate function F, which specifies the nonlinearity. These are known to enjoy conservation of energy/Hamiltonian and particle number

$$\mathscr{E}(u) = \frac{1}{2}\int_{\mathbb{R}^n} |\nabla u|^2 - \int_{\mathbb{R}^n} G(u^2(t,x))dx = \mathscr{E}(u_0), \mathscr{P}(u) = \int_{\mathbb{R}^n} |u(t,x)|^2 dx = \mathscr{P}(u_0)$$

where $G(0) = 0, G'(z) = F(z)$.

Furthermore, in may of these problems, there are soliton solutions. Namely for NLS, $e^{i\omega t}\varphi(x)$ and traveling waves for KdV equation $\varphi(x - \omega t)$. These satisfy non-linear elliptic PDE's and do not in general solve explicitly. Their properties are however crucial for the applications, that we have in mind. One of the central questions of interest is the stability of these waves.

To introduce this notion, one takes the ansatz $u(x,t) = e^{i\omega t}(\varphi + v(t,x))$ and expands in powers of v, by then ignoring $O(v^2)$ terms and a similar construction is applied to the traveling waves for KdV. The resulting linearized equation, say

$$v_t = \mathscr{A} v$$

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is studied in detail. The operator $\mathscr{A}$ is generally unbounded, and it has a number of important properties. The stability of the wave φ means absence of spectrum of $\mathscr{A}$ in the open right-half plane.

We discuss several recent results, pertaining to stability of said solitons. For example, for the related Hartree model - we present a full spectral characterization for all solitary waves, [1]. For the reduced Ostrovsky model, we construct traveling waves in the periodic case, and we identify the spectrally stable one, [2]. For the full Ostrovsky/short pulse model, in the whole line case, we construct and characterize traveling waves, [3]. Finally, we present a stability result for waves of the NLS model, with mixed power non-linearities, [4].

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