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MODERN THEMATIC PREPARATION FOR EIA IN MATHEMATICS IN UKRAINE: GEOMETRY *

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ABSTRACT: *In modern realities, the relevance of research on thematic preparation for the IEA in mathematics is undeniable. Based on the author's experience of systematization and repetition of the school mathematics course in preparation for IEA, we propose to divide the entire mathematics course into 10 logical content blocks: «Numbers and expressions», «Functions», «Equations and systems of equations», «Inequalities and systems of inequalities», «Text problems», «Elements of mathematical analysis», «Geometry on the plane», «Geometry in the space», «Coordinates and vectors», «Elements of combinatorics and stochastics».*

In this article, we provide thematic tests of the content blocks «Geometry on the plane» and «Geometry in the space», as well as answers to them. We also solve some of the basic tasks of these tests and give some methodical comments on these solutions. In the school course of mathematics geometric problems develop abstract logical thinking, spatial imagination and contribute to the formation of skills for practice tasks solving. We believe that a properly organized thematic systematization and repetition of the mathematics school course will allow students to successfully complete the IEA in mathematics and to help teachers achieve this success.

KEYWORDS: *IEA in mathematics, SFA in mathematics, thematic preparation, educational achievements of students, thematic tests, basic tasks, geometry.*

2020 Math. Subject Classification: *97D50 and 97D60, 97G40*

Formulation of the problem

External Independent Assessment (EIA) is now the main instrument of evaluation of the quality of mathematical training for

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Ukrainian senior school students. In particular, it is used for conducting the State Final Attestation (SFA) of academic achievements of graduates, as well as as a tool for competitive selection of applicants to Ukrainian high education institutions. Thus, we have no doubt about the importance and the need for research on various aspects of preparation for the EIA in mathematics. One such aspect is the thematic repetition of the school mathematics course.

Based on our experience in training students to EIA, during this repetition we divide the whole mathematics course into 10 thematic blocks: «Numbers and expressions», «Functions», «Equations and systems of equations», «Inequalities and systems of inequalities», «Text problems», «Elements of mathematical analysis», «Geometry on the plane», «Geometry in the space», «Coordinates and vectors», «Elements of combinatorics and stochastics». This division that allows repeated repetition of the same material throughout the preparation process for the EIA. For example, the system of coordinates are repeated during the study of thematic blocks 2, 3, 4, 6, 7, 8 and 9. This permits the teacher constantly to keep the student in a tone, when he or she would forget something, but can’t do this, because proposed thematic training system doesn’t allow it.

Analysis of current research

The problem of preparing students for EIA in mathematics is systematically regarded in different scientific and pedagogical papers. Constantly publish the results of their investigations in this area of research Valentyna Bevz, Mykhailo Burda, Hryhoriy Bilyanin, Olga Bilyanina, Olga Vashulenko, Larysa Dvoretska, Oxana Yergina, Oleksandr Ister, Vadym Karpik, Arkadiy Merzlyak, Yevgen Nelin, Victor Repeta, Oleksiy Tomaschuk, Mykhailo Yakir and others. During more then last 15 years, our author’s team has been constantly working to provide methodological support for the process of preparation for the EIA in mathematics. The theory and methodology of assessing the academic achievement of senior school students in Ukraine is given in the monograph [1]. For the training and systematization of the school mathematics course, we use the methodological set of books [2] and [3]. Previously, we have considered certain aspects of thematic preparation for independent testing, but since then the contingent of EIA participants has changed significantly, as well as the methodological views of our author's team on this problem are also developed. This article is the continuation of the series of our articles devoted to this problem that was started in the article [4].

The purpose of the article

The purpose of this article is to provide methodological advice to teachers and tutors that concerned in qualitative thematic preparation of graduates to EIA in mathematics. In particular, we present in this article two thematic tests related to the topics «Geometry on the plane» and «Geometry in the space», and also provide a solution of the some basic tasks of these tests with methodical comments to them.

Presentation of main material

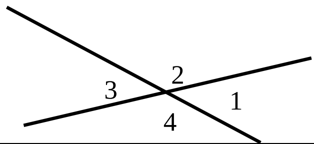
Geometry plays an extremely important role in the school course of mathematics. On the one hand, geometric objects have numerous practical applications in both everyday life and in many sciences. In particular, the ability to find areas of flat figures and body volumes allows students to adequately perceive tangible reality. Geometry, on the other hand, is perhaps the only school discipline that creates the child's abstract thinking. The axiomatic structure of geometry is similar to the logical structure of many formal systems, which include legal law, moral and ethical rules, religious and political constructs, and so on. In addition, the ability to make correct judgments from the right starting points to make the right conclusions is essential for any modern person.

For these reasons, geometric problems have to be presented in the math tests. At the same time, both application to practical tasks and testing the ability to justify abstract statements are important. In particular, open-ended tasks with full explanation serve the purpose of achieving the latter goal. We can be limited by tasks of this form in student's preparation to the EIA, because they are the most effective for teaching mathematics and feedback. However, after finishing each of the 10 thematic blocks, it is natural to perform a diagnostic thematic test in which to use all forms of test tasks inherent in the EIA math test. Let's look at two such tests below. The first refers to geometry on the plane and the second to geometry in space.

Thematic test «Geometry on the plane».

Tasks 1-7 have five answer choices, only one of which is correct. Choose the correct answer, in your opinion.

1. On the figure are drawn two intersecting straight lines, $\angle 1 + \angle 3 = 40°$. Define $\angle 2$.



A	B	C	D	E
$20°$	$70°$	$140°$	$160°$	$320°$

2. Find the catheter of a right-angled triangle if its two sides are equal to $\sqrt{5}$ and $\sqrt{14}$.

A	B	C	D	E
9	3	$\sqrt{14}+\sqrt{5}$	$\sqrt{14}-\sqrt{5}$	$\sqrt{19}$

3. Find the length of the midline of an equilateral trapezoid if the length of its side is 10 cm and the perimeter is 50 cm.

A	B	C	D	E
15 cm	20 cm	30 cm	35 cm	40 cm

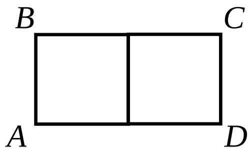
4. The sum of the two angles of the diamond is equal to $200°$. Find the sharp angle of this diamond.

A	B	C	D	E
50°	100°	160°	40°	80°

5. Specify the formula by which the area of a circle whose diameter is equal to d is calculated.

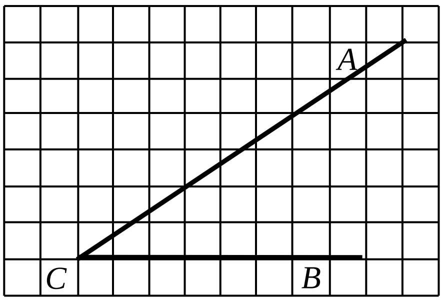
A	B	C	D	E
πd	$2\pi d$	$\frac{\pi d^2}{4}$	$\frac{\pi d^2}{2}$	πd^2

6. Rectangle $ABCD$ consists of two squares (see figure). Determine the perimeter of the rectangle $ABCD$, if the perimeter of each square equals $4a$.



A	B	C	D	E
$2a$	$4a$	$6a$	$7a$	$8a$

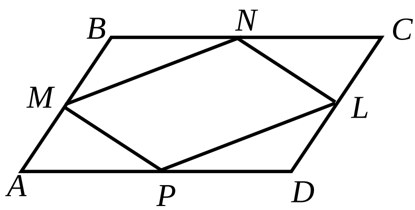
7. Determine the tangent of the angle ACB shown on the figure, drawn on the cell paper. Please give the answer that is closest to correct.



A	B	C	D	E
$\frac{2}{3}$	$\frac{1}{2}$	1	2	$\frac{3}{2}$

In the task 8 for each of the three rows of data marked with numbers, select the one correct, in your opinion, variant marked with a letter.

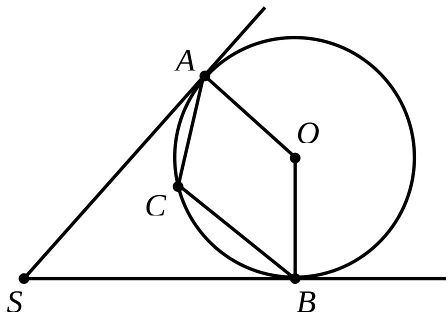
8. On the figure is shown a parallelogram $ABCD$, whose diagonals are 6 and 8. Points M, N, L, P are in the middle of the parties AB, BC, CD, AD respectively. Match the beginning of the sentence (1 – 3) and its end (A – E) so that the correct statement will be formed.



Beginning of the sentence	*End of the sentence*
1 The length of the segment AC is equal	**A** 14
2 The length of the segment MP is equal to	**B** 8
3 The perimeter of quadrilateral $MNLP$ is equal to	**C** 7
	D 6
	E 3

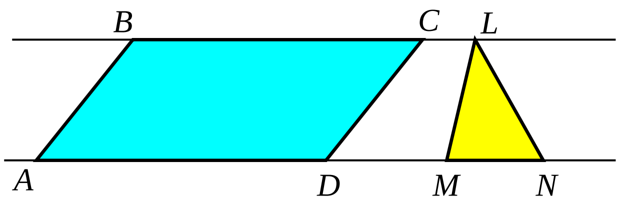
Solve tasks 9-11. Record the numeric answers you received *in decimal or integer*.

9. On the schematic illustration is shown a point S, a circle centered at a point O and two tangents SA and SB, drawn from the point S to the circle. Points A, B, C are on the circle. It is known that $\angle ACB = 110^{\circ}$. Find the degrees of angles:



1) $\angle AOB$; 2) $\angle ASB$.

10. The vertices of the parallelogram $ABCD$ and triangle MLN are on the straight parallel lines (see figure). The area of the parallelogram $ABCD$ is equal to 546 cm^2, length of the segment AD is three times the length of the segment MN.



Find the area of the triangle MLN (in cm^2).

11. The lengths of the sides of a regular hexagon and a square are the same. Find the ratio of the area of the circle inscribed in the square to the area of the circle described around the hexagon.

Solve the task 12. Write down sequential logical actions and explanations of all stages of task solving, make reference to the mathematical facts from which one or another statement follows. If necessary, illustrate the task solving with drawings, graphs, etc.

12. The triangle ABC is isosceles with the base BC, $\angle A = \alpha$. The radius of the circle described around the triangle ABC is equal to R, and the center of this circle is outside the triangle ABC. Prove that $\alpha > 90^{\circ}$. Find all other sides and angles of the triangle ABC and also radius r of the circle inscribed in it.

Answers to the test «Geometry on the plane»

1	2	3	4	5	6	7	8	9	10	11
D	B	A	E	C	C	A	1 – B, 2 – E, 3 – A	1) 140; 2) 40	91	0,25

12. $\angle B = \angle C = 90^{\circ} - \dfrac{\alpha}{2}$, $AB = AC = 2R\cos\frac{\alpha}{2}$, $BC = 2R\sin\alpha$,

$$r = \frac{2R\sin\alpha\cos^{2}\frac{\alpha}{2}}{\sin\alpha + 2\cos\frac{\alpha}{2}}.$$

Solutions and comments to tasks of the test «Geometry on the plane».

Task 9 (term of the task see above). *Solution.* 1) According to the property of the angle inscribed in the circle, its degree measure is equal to half the degree measure of the *flat angle* corresponding to it. Therefore, the degree measure of the needed angle is equal to $360^{\circ} - 2\cdot 110^{\circ} = 140^{\circ}$. 2) It is known that the sum of the angles of the convex quadrilateral *SAOB* is 360 degrees. Since *SA* and *SB* are tangent to the circle, then $\angle SAO = SBO = 90^{\circ}$ and $\angle AOB = 140^{\circ}$. Thus, $\angle ASB = 360^{\circ} - 2\cdot 90^{\circ} - 140^{\circ} = 40^{\circ}$.

Comment. In this task, it is important to focus students' attention on the differences between the concepts of *angle* and *flat angle*, which in the school mathematics traditionally are called by one term «angle». Similar arrangements in geometry are quite common: for example, the radius of a circle and its length are also denoted by one term «radius». Also in this task, the teacher should explain that the central flat angle corresponding to a given inscribed angle is bounded by the arc on which the vertex of the inscribed angle *does not* lie.

Task 12 (term of the task see above). *Solution.* If through the center of the circle described around the triangle draw the diameter parallel to its base, then the angle inscribed in this circle with the vertex at point *A* will be straight. The angle α is also inscribed in the same circle and has the same vertex, but relies on a larger arc.

Therefore, its degree is greater than 90 degrees that was needed to prove.

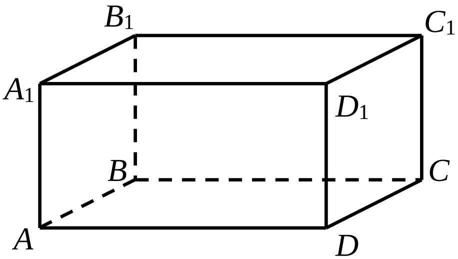
Since the sum of the angles of a triangle is 180 degrees and the angles at the base of an isosceles triangle are equal, then $\angle B = \angle C = 90^{\circ} - \frac{\alpha}{2}$. By the sine theorem $\frac{BC}{\sin\alpha} = \frac{AB}{\sin\angle C} = \frac{AC}{\sin\angle B} = 2R$, whereof $AB = AC = 2R\cos\frac{\alpha}{2}$ and $BC = 2R\sin\alpha$. To find the radius of the inscribed circle we use the formula $r = \frac{2S}{AB + BC + AC}$, where S is the triangle area. Because $S = \frac{1}{2} AC \cdot AB \cdot \sin\alpha$, then $r = \frac{2R\sin\alpha\cos^2\frac{\alpha}{2}}{\sin\alpha + 2\cos\frac{\alpha}{2}}$ and task solving is completed.

Comment. The solution to this problem consists of two parts – proving some statement and using known formulas and theorems to calculate unknown elements. It is very important even in the computational part not only the use of formulas, but also references to the corresponding theorems and construction for a clear logical chain from condition to result. Such task with full explanation in the EIA test is estimated at 4 points. For Task 12, the student receives the first point for a complete and correct justification that the angle α is greater than 90 degrees. He receives another point for finding the angles at the base of the triangle. Another point is for finding the sides from the sine theorem. Finally, the student receives the last point for finding the radius of the inscribed circle.

Thematic test «Geometry in the space».

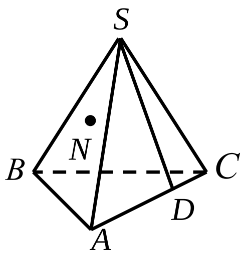
Tasks 1-7 have five answer choices, only one of which is correct. Choose the correct answer, in your opinion.

1. On the figure is drawn a rectangular parallelepiped $ABCDA_1B_1C_1D_1$. Specify the straight line that belongs to the plane (ADD_1).



A	B	C	D	E
AB	AC	BD	D_1C	A_1D

2. Point N is on the facet SAB of tetrahedron $SABC$, point D is on the segment AC (see figure). Specify a straight line that is the intersection of the plane SAB and the plane passing through the straight line SD and point N.



A	B	C	D	E
BN	SN	AN	SD	SA

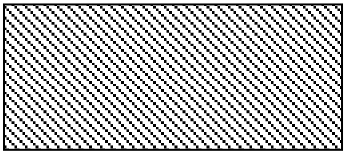
3. By which formula the area of full surface S of the regular rectangular pyramid whose side of the base is equal to a and the apophemus is equal to L is calculated?

A	B	C	D	E
$S=a^2+2aL$	$S=a^2L$	$S=2aL$	$S=\dfrac{a^2L}{3}$	$S=2a^2+2aL$

4. In a cone whose radius of the base is equal to r and the axial section is a right triangle with the side l *inscribed* the ball with radius R. Specify the correct equality.

A	B	C	D	E
$R=r$	$R=\dfrac{l}{2\sqrt{3}}$	$R=\dfrac{r}{2\sqrt{3}}$	$R=\dfrac{l}{\sqrt{3}}$	$R=2r$

5. On the figure is shown a *side sweep* of a body. Specify this body.



A	B	C	D	E
Triangular pyramid	Quadrangular pyramid	Ball	Cylinder	Cone

6. The heights of the cylinder and the cone are equal, and the radius of the base of the cone is 6 times larger than the radius of the base of the cylinder. Find the ratio of cylinder volume to cone volume.

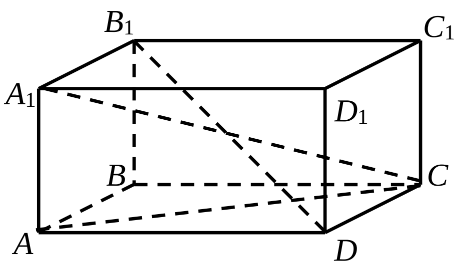
A	B	C	D	E
1:36	1:6	1:12	1:3	1:2

7. Ball with radius $R=15$ cm crossed the plane, which is 9 cm away from the center of the ball. Find the area of the section formed (in cm2).

A	B	C	D	E
225π	18π	144π	24π	81π

In the task 8 for each of the three rows of data marked with numbers, select the one correct, in your opinion, variant marked with a letter.

8. On the picture is shown a rectangular parallelepiped $ABCDA_1B_1C_1D_1$ Match the beginning of the sentence (1 – 3) and its end (A – E) so that the correct statement will be formed.



Beginning of the sentence

1 Straight lines CA_1 and DB_1

2 Straight lines CA_1 and AC

3 Straight lines CA and DB_1

End of the sentence

A are intersected at a point that belongs to the plane (ABA_1)

B are intersected at a point that belongs to the plane (DBD_1)

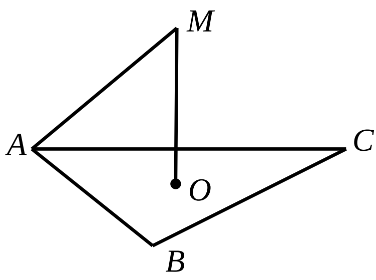
C are intersected at a point that belongs to the plane (ABC)

D are parallel

E are crossbreeding

Solve tasks 9-11. Record the numeric answers you received *in decimal or integer*.

9. Through the center O of a right triangle ABC is drawn a segment MO that perpendicular to the plane ABC (see figure). The area of this triangle is equal to $48\sqrt{3}$ cm and $MA = 10$ cm. Calculate (in cm):



   1) radius of the circle described around the triangle ABC;

   2) distance from the point M to the plane ABC.

10. The assembly hall of the school has the form of a rectangular parallelepiped, the dimensions of which are 10m, 21m and 5m. To install individual heating in this room, one plan to use the same gas convectors, each of which is designed for heating for 150m3 of air. How many convectors are needed to install?

11. The lateral surface of the cone was expanded into a sector. Find the degree measure of the center angle α of this sector if the ratio of the diameter of the base of the cone to the generating cone is equal to $4:3$.

Solve the task 12. Write down sequential logical actions and explanations of all stages of task solving, make reference to the mathematical facts from which one or another statement follows. If necessary, illustrate the task solving with drawings, graphs, etc.

12. A straight quadrilateral prism $ABCDA_1B_1C_1D_1$, the base of which is a diamond with a side a and a sharp angle

$BAD = \alpha$ is given. The volume of the prism is equal to V. 1) Find the area S of the lateral surface of the prism. 2) Construct by the method of traces the cross section of this prism that runs through the top D the lower base and the middle of the edge A_1B_1 and B_1C_1 of the upper base. Determine what geometric figure this section is. 3) Find the angle β between the side edge DD_1 and the cross-section plane.

Answers to the test «Geometry in the space»

1	2	3	4	5	6	7	8	9	10	11
E	B	A	B	D	C	C	1 – B, 2 – C, 3 – E	1) 8; 2) 6	7	240

12. 1) $S = \dfrac{4V}{a\sin\alpha}$. 2) The cross section is a pentagon. 3) $\beta = \operatorname{arctg}\left(\dfrac{3a^3\sin\alpha\sin\frac{\alpha}{2}}{2V}\right)$.

Solutions and comments to tasks of the test «Geometry in the space».

Task 8 (term of the task see above). *Solution.* **1**. These straight lines are intersect at a point belonging to the plane (DBD_1), that completely contains the straight line DB_1. **2**. These straight lines are intersect at the point C that belongs to the plane (ABC). **3**. These straight lines have no common point, but do not belong to the same plane. Thus, they are crossbreeding lines. So, the correct answer is **1** – **B**, **2** – **C**, **3** – **E**.

Comment. This task checks the formation of students' spatial imagination and is not technically difficult. However, in order to find the right answer, it is also important not only to use intuition, but to prove the correctness of each statement that was used. This is the way to reduce the number of errors during solving of such tasks.

Task 10 (term of the task see above). *Solution.* The volume of the assembly hall is equal to $10 \cdot 21 \cdot 5 = 1050\,\text{m}^3$. Since one gas convector is designed to heat 150 m^3 of air, then $1050 : 150 = 7$ such convectors are required.

Comment. This task is also not technically difficult, but it does not test the theoretical knowledge, but the students' ability to create a mathematical model of real processes and phenomena. It is clear that the above task is educational and in reality, almost never, as a result of division, we will not get an integer. Therefore, after solving task 10, it is advisable to offer children a series of similar tasks that require additional conditions. For example, if each gas convector is designed to heat only 100 m^3 of air, you first need to find out whether the overheating of the room is more harmful than a slight underheating. Depending on the answer to this question, the answer will be either 10 or 11 gas convectors needed.

Conclusions

The role of geometry in shaping students' abstract thinking is enormous. In fact, only geometry is the only discipline at school that can achieve this goal. Therefore, geometric problems require proper attention during preparing to SFA and EIA. It is up to the teacher to be aware of their importance and to convey this importance to children. At the same time, their attention should be paid to the fact that the main thing in geometry is the proper reasoning of all logical steps for solving of every problem.

We believe that well-organized thematic training for EIA and SFA in mathematics will allow teachers to overcome the problems encountered by students in the systematization and repetition of the school mathematics course. We hope that the offered methodological tips will be useful for all teachers involved in this process. In future publication, we plan to complete the series of our publications, devoted to repetition features for all of the above thematic blocks with providing the summary test, solutions to the basic tasks and with providing of methodological comments to them.

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