MERGING OF BIVARIATE COMPOUND POISSON RISK PROCESSES WITH SHOCKS*

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ABSTRACT: *The paper considers an example of reducing a bivariate Compound Poisson risk process with common shocks to an univariate Cramer-Lundberg risk model. Then using the results about Compound Poisson risk model the main numerical characteristics and distributions related to the new model are obtained. Analogous approach can be applies for any number of compound Poisson processes. It can be useful also in renewal theory or queuing theory for simplifying systems or building stochastically equivalent models.*

KEYWORDS: *Multivariate risk models, Common shocks, Cramer-Lundberg risk model.*

1 Introduction

The theory of Compound Poisson risk processes origins since the works of Filip Lundberg [9] and Harald Cramer [3] and finds good applications nowadays. In 1967 Marshall and Olkin [10], [11] investigate a bivariate Poisson model with dependent coordinates. They lay the foundations of multivariate models with common shocks. Particular cases of such models are considered for example in Cossette and Marceau [2], or Wand [19]. In any of them the task for characterising of total claim amount distribution reduces to calculations of multivariate compound distributions. Recursive algorithms for bivariate compound distributions of Panjer's family are obtained e.g. in Panjer and Willmot[13, 14], Hesselager [12] among others. Sundt and Vernic [16, 17, 18] generalize these recursions to the multivariate case. Recently Jordanova and Stehlik [8] show that in many cases these multivariate risk models can be reduced to classical Cramer-Lundberg model. Here we consider an example of merging of Compound Poisson processes. More general results could be seen in Jordanova and Stehlik [8].

The organization of the paper is the following...

Along the paper $\perp$ means independent, $HPP(\lambda)$ is an abbreviation of "homogeneous Poisson process with intensity $\lambda > 0$", $g_\xi(x) = Ez^\xi$ is the probability generating function of the random variable ξ and $\sum_{i=1}^{0} = 0$. The independence between the summands and the number of summands in the following random sums can be relaxed without changing the final results if we assume that the compounding distributions describe the distribution of the summands, given the number of summands.

2 Description of the model

Consider a risk process R consisting of two dependent types of polices. The dependent counting processes M_1 and M_2 of both types are compound Poisson process $B_k = \{B_k(t) : t \geq 0\}$, $k = 1,2$ with common shocks, described by independent compound Poisson process $B_0 = \{B_0(t) : t \geq 0\}$. More precisely for all $t \geq 0$

(1) $$M_1(t) = B_1(t) + B_0(t), \quad M_2(t) = B_2(t) + B_0(t),$$

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where the processes B_0, B_1 and B_2 are independent, and for $k = 0,1,2$,

$$B_k(t) = \sum_{i=1}^{N_k(t)} \eta_{ki},\ N_k \sim HPP(\lambda_k),\ g_k(z) = Ez^{\eta_{k1}}, z \geq 0, \quad \{\eta_{ki}\}_{i=1}^{\infty} \perp N_k, \tag{2}$$

$\eta_{k1}, \eta_{k2}, \ldots$ independent identically distributed (i.i.d.), and $\eta_{11} \perp \eta_{21} \perp \eta_{01}$.

$S = \{S(t) : t \geq 0\}$ is the total claim amount process. For all $t \geq 0$

$$S(t) = S_1(t) + S_2(t), \tag{3}$$

$$S_1(t) = \sum_{i=1}^{B_1(t)} Y_{1i} + \sum_{i=1}^{B_0(t)} Y_{0i}, \quad \{Y_{ki}\}_{i=1}^{\infty} \perp B_i,\ k = 0,1, \tag{4}$$

$$S_2(t) = \sum_{i=1}^{B_2(t)} Y_{2i} + \sum_{i=1}^{B_0(t)} Y_{0i}, \quad \{Y_{ki}\}_{i=1}^{\infty} \perp B_i,\ k = 0,2.$$

$Y_{k1}, Y_{k2}, \ldots$ i.i.d., and $Y_{01} \perp Y_{11} \perp Y_{21}$.

$R = \{R(t) : t \geq 0\}$ is the risk reserve process and for all $t \geq 0$

$$R(t) = u + ct - S(t), \tag{5}$$

where $u \geq 0$ is the initial capital and $c > 0$ is the premium income rate.

3 Stochastic equivalent models

In this section first we obtain a stochastically equivalent presentation of the bivariate counting process $\{(M_1(t), M_2(t)) : t \geq 0\}$ which shows that it is a particular case of multivariate compound Poisson processes of type I, discussed in Sundt and Vernic [17, 18]. Then, in Theorem 2 we derive the stochastic equivalent presentation of the processes (S_1, S_2) as a compound Poisson process of Type I. Finally, in Theorem 3 we describe a stochastic equivalence presentation of the risk process R which allows us to state that this is a Cramer-Lundberg (C-L) risk process.

Denote by $\lambda = \lambda_1 + \lambda_2 + \lambda_0$. In order to formulate our main results let us define a random vector (I_1, I_2, I_0) with probability mass function (p.m.f.)

$$P(I_1 = 1, I_2 = 0, I_0 = 0) = \frac{\lambda_1}{\lambda},\ dP(I_1 = 0, I_2 = 1, I_0 = 0) = \frac{\lambda_2}{\lambda},\ P(I_1 = 0, I_2 = 0, I_0 = 1) = \frac{\lambda_0}{\lambda}, \tag{6}$$

and zero otherwise. Assume random vectors $(I_{11}, I_{21}, I_{01}), (I_{12}, I_{22}, I_{02}), \ldots$ are i.i.d. with p.m.f. (6) and $(\eta_{11}, \eta_{21}, \eta_{01}), (\eta_{12}, \eta_{22}, \eta_{02}), \ldots$ are i.i.d. with independent coordinates with probability generating functions (p.g.fs.) correspondingly g_1, g_2, g_0. Denote by $N \sim HPP(\lambda)$,

$$A_1(t) = \sum_{i=1}^{N(t)} (I_{1i}\eta_{1i} + I_{0i}\eta_{0i}), \tag{7}$$

$$A_2(t) = \sum_{i=1}^{N(t)} (I_{2i}\eta_{2i} + I_{0i}\eta_{0i}). \tag{8}$$

Assume N, $(I_{11}, I_{21}, I_{01}), (I_{12}, I_{22}, I_{02}), \ldots$ and $(\eta_{11}, \eta_{21}, \eta_{01}), (\eta_{12}, \eta_{22}, \eta_{02}), \ldots$ are independent.

Theorem 1. The bivariate processes (M_1, M_2) and (A_1, A_2) coincide in the sense of their finite dimensional distributions.

Proof: All processes (M_1, M_2) and (A_1, A_2) have homogeneous and independent additive increments and they start from the coordinate beginning, therefore in order to prove their stochastic equivalence it is enough to prove equality of their univariate time intersections. Due to the uniqueness of the correspondence between the probability laws and their p.g.fs. it is enough to derive equality between the p.g.fs. Consider $t > 0$, $z_1 > 0$ and $z_2 > 0$. The definition and the multiplicative property of p.g.fs., (1), (2), and the well known form of the p.g.f. of compound Poisson processes imply

$$\begin{aligned}
E\left[z_1^{M_1(t)} z_2^{M_2(t)}\right] &= E\left[z_1^{B_1(t)+B_0(t)} z_2^{B_2(t)+B_0(t)}\right] = E\left[z_1^{B_1(t)}\right] E\left[z_2^{B_2(t)}\right] E\left[(z_1 z_2)^{B_0(t)}\right] \\
&= E\left[z_1^{\sum_{i=1}^{N_1(t)} \eta_{1i}}\right] E\left[z_2^{\sum_{i=1}^{N_2(t)} \eta_{2i}}\right] E\left[(z_1 z_2)^{\sum_{i=1}^{N_0(t)} \eta_{0i}}\right] \\
&= exp\{-\lambda_1 t[1-g_1(z_1)]\} exp\{-\lambda_2 t[1-g_2(z_2)]\} exp\{-\lambda_0 t[1-g_0(z_1 z_2)]\} \\
&= exp\{-(\lambda_1+\lambda_2+\lambda_0)t - \lambda_1 t g_1(z_1) - \lambda_2 t g_2(z_2) - \lambda_0 t g_0(z_1 z_2)\} \\
&= exp\left\{-(\lambda_1+\lambda_2+\lambda_0)t\left[1-\frac{\lambda_1}{\lambda} g_1(z_1) - \frac{\lambda_2}{\lambda} g_2(z_2) - \frac{\lambda_0}{\lambda} g_0(z_1 z_2)\right]\right\}.
\end{aligned}$$

The definitions (7), (8), the formula for double expectations, the independence between N and the other components of the processes A_1 and A_2 entail

$$\begin{aligned}
E\left[z_1^{A_1(t)} z_2^{A_2(t)}\right] &= E\left[z_1^{\sum_{i=1}^{N(t)}(I_{1i}\eta_{1i}+I_{0i}\eta_{0i})} z_2^{\sum_{i=1}^{N(t)}(I_{2i}\eta_{2i}+I_{0i}\eta_{0i})}\right] \\
&= \sum_{n=0}^{\infty} E\left[z_1^{\sum_{i=1}^{n}(I_{1i}\eta_{1i}+I_{0i}\eta_{0i})} z_2^{\sum_{i=1}^{n}(I_{2i}\eta_{2i}+I_{0i}\eta_{0i})}\right] P(N(t)=n)
\end{aligned}$$

By the multiplicative property of p.g.fs. and (6),

$$E\left[z_1^{A_1(t)} z_2^{A_2(t)}\right] = \sum_{n=0}^{\infty} \left\{E\left[z_1^{I_{1i}\eta_{11}+I_{01}\eta_{01}} z_2^{I_{21}\eta_{21}+I_{01}\eta_{01}}\right]\right\}^n P(N(t)=n). \tag{9}$$

Now (6) and the total probability formula imply

$$E\left[z_1^{I_{1i}\eta_{11}+I_{01}\eta_{01}} z_2^{I_{21}\eta_{21}+I_{01}\eta_{01}}\right] = \frac{\lambda_1}{\lambda} g_1(z_1) + \frac{\lambda_2}{\lambda} g_2(z_2) + \frac{\lambda_0}{\lambda} g_0(z_1 z_2). \tag{10}$$

Therefore the fact that $N(t) \sim HPP(\lambda t)$ entails

$$\begin{aligned}
E\left[z_1^{A_1(t)} z_2^{A_2(t)}\right] &= \sum_{n=0}^{\infty} \left\{\frac{\lambda_1}{\lambda} g_1(z_1) + \frac{\lambda_2}{\lambda} g_2(z_2) + \frac{\lambda_0}{\lambda} g_0(z_1 z_2)\right\}^n P(N(t)=n) \\
&= exp\left\{-\lambda t\left[1-\frac{\lambda_1}{\lambda} g_1(z_1) + \frac{\lambda_2}{\lambda} g_2(z_2) + \frac{\lambda_0}{\lambda} g_0(z_1 z_2)\right]\right\}
\end{aligned}$$

which is exactly $E\left[z_1^{M_1(t)} z_2^{M_2(t)}\right]$, and completes the proof.

Theorem 2. The bivariate total claim amount processes (S_1, S_2), described in (5) is a compound poisson Process of type I (which means with one and the same number of summands). It is stochastically equivalent to the process (S_3, S_4), where

$$S_3(t) = \sum_{n=1}^{N(t)} \left[I_{1n} \sum_{i=1}^{\eta_{1n}} Y_{1i} + I_{0n} \sum_{i=1}^{\eta_{0n}} Y_{0i} \right], \quad t \geq 0, \tag{11}$$

$$S_4(t) = \sum_{n=1}^{N(t)} \left[I_{2n} \sum_{i=1}^{\eta_{2n}} Y_{2i} + I_{0n} \sum_{i=1}^{\eta_{0n}} Y_{0i} \right], \quad t \geq 0, \tag{12}$$

where (I_{11}, I_{21}, I_{01}) is a vector with distribution (6) and the other random elements are the same as those described in (6), (7) and (8). The equivalence is in the sense of their finite dimensional distributions.

Proof: All processes have homogeneous and independent additive increments and they start from the coordinate beginning, therefore in order to prove their stochastic equivalence it is enough to prove equality of their univariate time intersections. Due to the uniqueness of the correspondence between the probability laws and their Laplace-Stieltjes transforms(LSTs), it is enough to derive equality between the (LSTs).

Consider $t \geq 0$, $z_1 \geq 0$ and $z_2 \geq 0$. The definition of LSTs., (5), and the multiplicative property of LSTs, the formula for relation between the LST of compound distribution and LST of summands and the number of summands, the well known form of the LST of compound Poisson processes, (2), (1) imply

$$\begin{aligned}
Ee^{-z_1 S_1(t) - z_2 S_2(t)} &= Ee^{-z_1 \sum_{i=1}^{B_1(t)} Y_{1i} - (z_1+z_2) \sum_{i=1}^{B_0(t)} Y_{0i} - z_2 \sum_{i=1}^{B_2(t)} Y_{2i}} \\
&= Ee^{-z_1 \sum_{i=1}^{B_1(t)} Y_{1i}} Ee^{-(z_1+z_2) \sum_{i=1}^{B_0(t)} Y_{0i}} Ee^{-z_2 \sum_{i=1}^{B_2(t)} Y_{2i}} \\
&= g_{B_1(t)}(Ee^{-z_1 Y_{11}}) g_{B_0}[Ee^{-(z_1+z_2) Y_{01}}] g_{B_2(t)}(Ee^{-Y_{21}}) \\
&= e^{-\lambda_1 t [1 - g_1(Ee^{-z_1 Y_{11}})]} e^{-\lambda_0 t \left\{1 - g_0[Ee^{-(z_1+z_2) Y_{01}}]\right\}} e^{-\lambda_2 t [1 - g_2(Ee^{-z_2 Y_{21}})]} \\
&= e^{-(\lambda_1+\lambda_2+\lambda_3) t \left\{1 - \left[\frac{\lambda_1}{\lambda} g_1(Ee^{-z_1 Y_{11}}) + \frac{\lambda_0}{\lambda} g_0[Ee^{-(z_1+z_2) Y_{01}}] + \frac{\lambda_2}{\lambda} g_2(Ee^{-z_2 Y_{21}})\right]\right\}}.
\end{aligned}$$

From the other side if we consider $z_3 \geq 0$ and $z_4 \geq 0$, the definitions (11), (12), the formula for double expectations, the independence between N and the other components of the processes S_3 and S_4 entail

$$\begin{aligned}
Ee^{-z_3 S_3(t) - z_4 S_4(t)} &= Ee^{-z_3 \sum_{n=1}^{N(t)} \left[I_{1n} \sum_{i=1}^{\eta_{1n}} Y_{1i} + I_{0n} \sum_{i=1}^{\eta_{0n}} Y_{0i}\right] - z_4 \sum_{n=1}^{N(t)} \left[I_{2n} \sum_{i=1}^{\eta_{2n}} Y_{2i} + I_{0n} \sum_{i=1}^{\eta_{0n}} Y_{0i}\right]} \\
&= \sum_{n=0}^{\infty} \left\{ Ee^{-z_3 I_{11} \sum_{i=1}^{\eta_{11}} Y_{1i} - (z_3+z_4) I_{01} \sum_{i=1}^{\eta_{01}} Y_{0i} - z_4 I_{21} \sum_{i=1}^{\eta_{21}} Y_{2i}} \right\}^n P(N(t) = n).
\end{aligned}$$

Now (6) and the total probability formula and the formula about the relation between the for LST of a compound, the LST of the summands and p.g.f. of the number of summands, imply

$$Eexp\left\{ -z_3 I_{11} \sum_{i=1}^{\eta_{11}} Y_{1i} - (z_3+z_4) I_{01} \sum_{i=1}^{\eta_{01}} Y_{0i} - z_4 I_{21} \sum_{i=1}^{\eta_{21}} Y_{2i} \right\} =$$

$$\begin{aligned}
&= \frac{\lambda_1}{\lambda} Ee^{-z_3 \sum_{i=1}^{\eta_{11}} Y_{1i}} + \frac{\lambda_0}{\lambda} Ee^{-(z_3+z_4) \sum_{i=1}^{\eta_{01}} Y_{0i}} + \frac{\lambda_2}{\lambda} Ee^{-z_4 \sum_{i=1}^{\eta_{21}} Y_{2i}} \\
&= \frac{\lambda_1}{\lambda} g_1(Ee^{-z_3 Y_{11}}) + \frac{\lambda_0}{\lambda} g_2(Ee^{-(z_3+z_4) Y_{01}}) + \frac{\lambda_2}{\lambda} g_2(Ee^{-z_4 Y_{21}})
\end{aligned}$$

Therefore the formula for p.g.f. of Poisson distribution entails

$$
\begin{aligned}
Ee^{-z_3S_3(t)-z_4S_4(t)} &= \sum_{n=0}^{\infty}\left\{\frac{\lambda_1}{\lambda}g_1(Ee^{-z_3Y_{1i}})+\frac{\lambda_0}{\lambda}g_2(Ee^{-(z_3+z_4)Y_{0i}})+\frac{\lambda_2}{\lambda}g_2(Ee^{-z_4Y_{2i}})\right\}^n P(N(t)=n) \\
&= e^{-\lambda t\left\{1-\left[\frac{\lambda_1}{\lambda}g_1(Ee^{-z_3Y_{11}})+\frac{\lambda_0}{\lambda}g_0[Ee^{-(z_3+z_4)Y_{01}}]+\frac{\lambda_2}{\lambda}g_2(Ee^{-z_4Y_{21}})\right]\right\}}.
\end{aligned}
$$

which is exactly $Ee^{-z_1S_1(t)-z_2S_2(t)}$. In this way the proof is completed.

Now we are ready to say that the risk process R, defined in (5) is a C-L risk process.

Theorem 3. The risk process, described in (5) is stochastically equivalent to the process

$$
R(t)=u+ct-\sum_{n=1}^{N(t)}\left[I_{1n}\sum_{i=1}^{\eta_{1n}}Y_{1i}+2I_{0n}\sum_{i=1}^{\eta_{0n}}Y_{0i}+I_{2n}\sum_{i=1}^{\eta_{2n}}Y_{2i}\right], \quad t\geq 0.
$$

The proof of Theorem 3 follows immediately from the definition (5) of the risk process R, the definitions of S, S_1 and S_2, (see 4, and 5) and Theorem 2.

4 The characteristics of the risk model R

In this section using the well known results about the C-L risk model we obtain the numerical characteristics, conditional distributions and probabilities for ruin of the risk model (5).

Denote by

$\tau(u)\ = inf\{t>0 : R(t)<0|R(0)=u\}, u\geq 0, inf=\infty$, the time of ruin with initial capital $u\geq 0$;

$\psi(u)\ = P(\tau(u)<\infty|R(0)=u)$, the probability for ruin in infinite horizon;

$\delta(u)\ = 1-\psi(u)$, the probability for survival in infinite horizon;

$\lambda\ = \lambda_1+\lambda_2+\lambda_3$ is the intensity of the merged $N(t)\sim HPP(\lambda)$. Then it is the parameter of the exponential distribution, which models the inter-arrival times in the merged risk process presentation.

The mixed claim size in the merged risk process presentation are

$$
Y_s := I_{1s}\sum_{i=1}^{\eta_{1s}}Y_{1i}+2I_{0s}\sum_{i=1}^{\eta_{0s}}Y_{0i}+I_{2s}\sum_{i=1}^{\eta_{2s}}Y_{2i}, \quad s=1,2,...
$$

Note that these r.vs. are i.i.d.

In the next theorem using the results for the Cramer-Lundberg model (see e.g. Grandell [6], Gerber [5], Embrechts, Klueppelberg, Mikosch [4] Rolski, Schmidli, Schmidt, Teugels [15]) we characterise the risk process R.

Theorem 4. Consider the risk model R, defined in (5). Denote by

$$
\mu=\lambda_1E(\eta_{11})E(Y_{11})+2\lambda_0E(\eta_{01})E(Y_{01})+\lambda_2E(\eta_{21})E(Y_{21}).
$$

Then

i) In case when the expectations are finite, the safety loading is $\rho=\frac{c}{\mu}-1$ and the net profit condition is $c>\mu$.

ii) In case when the expectations are finite and the net profit is satisfied $\psi(0) = \frac{\mu}{c}$.

iii) The general formula for the Laplace - Stieltjes transform of $\delta(u)$ is

$$l_\delta(s) = \frac{s(c-\mu)}{cs - \lambda - \lambda_1 g_1(Ee^{-sY_{11}}) - \lambda_2 g_2(Ee^{-sY_{21}}) - \lambda_0 g_0(Ee^{-2sY_{01}})}.$$

iv) The distribution of the deficit at the time of ruin, given that ruin with initial capital zero happen has the following distribution

$\mathbb{P}(-R(\tau(0)+) \leq x | \tau(0) < \infty) =$

$$= \sum_{k=1}^{2} \left\{ \frac{\lambda_k}{\mu} \int_0^x P\left(\sum_{i=1}^{\eta_{k1}} Y_{k1} \geq y \right) dy \right\} + \frac{\lambda_0}{\mu} \int_0^x P\left(\sum_{i=1}^{\eta_{01}} Y_{01} \geq \frac{y}{2} \right) dy.$$

v) Given the second moments exist,

$\mathbb{E}(-R(\tau(0)+) | \tau(0) < \infty) =$

$$= \frac{1}{\mu} \left\{ \sum_{k=1}^{2} \lambda_k \left[E(\eta_{k1}) D(Y_{k1}) + E(\eta_{k1}^2) [E(Y_{k1})]^2 \right] + 4\lambda_0 \left[E(\eta_{01}) D(Y_{01}) + E(\eta_{01}^2) [E(Y_{01})]^2 \right] \right\}.$$

$\mathbb{E}(\tau(0) | \tau(0) < \infty) =$

$$= \frac{\sum_{k=1}^{2} \lambda_k \left[E(\eta_{k1}) D(Y_{k1}) + E(\eta_{k1}^2) [E(Y_{k1})]^2 \right] + 4\lambda_0 \left[E(\eta_{01}) D(Y_{01}) + E(\eta_{01}^2) [E(Y_{01})]^2 \right]}{\mu(c-\mu)}.$$

vi) The joint distribution of the severity of (deficit at) ruin and the risk surplus just before the ruin with initial capital zero is:

$\mathbb{P}(-R(\tau(0)+) > x, R(\tau(0)-) > y | \tau(0) < \infty) =$

$$= \sum_{k=1}^{2} \left\{ \frac{\lambda_k}{\mu} \int_0^{x+y} P\left(\sum_{i=1}^{\eta_{k1}} Y_{k1} \geq y \right) dy \right\} + \frac{\lambda_0}{\mu} \int_0^{x+y} P\left(\sum_{i=1}^{\eta_{01}} Y_{01} \geq \frac{y}{2} \right) dy$$

vii) The distribution of the claim causing ruin is

$\mathbb{P}(R(\tau(0)-) - R(\tau(0)+) \leq x | \tau(0) < \infty) =$

$$= \sum_{s=1}^{2} \frac{\lambda_s}{\mu} \int_0^x y dP\left(\sum_{i=0}^{\eta_{s1}} Y_{s1} < y\right) + \frac{2\lambda_0}{\mu} \int_0^{\frac{x}{2}} y dP\left(\sum_{i=0}^{\eta_{01}} Y_{01} < y\right).$$

viii) If the distribution of Y_1 is domilatetly varying which means that

$$\limsup_{x\to\infty} \frac{\lambda_1 P(\sum_{i=0}^{\eta_{11}} Y_{11} < \frac{x}{2}) + \lambda_2 P(\sum_{i=0}^{\eta_{21}} Y_{21} < \frac{x}{2}) + \lambda_0 P(\sum_{i=0}^{\eta_{01}} Y_{01} < \frac{x}{4})}{\lambda_1 P(\sum_{i=0}^{\eta_{11}} Y_{11} < x) + \lambda_2 P(\sum_{i=0}^{\eta_{21}} Y_{21} < x) + \lambda_0 P(\sum_{i=0}^{\eta_{01}} Y_{01} < \frac{x}{2})} < \infty,$$

then

$$\lim_{u\to\infty} \frac{\psi(u)}{\int_0^u \left[\sum_{k=1}^{2} \lambda_k P\left(\sum_{i=1}^{\eta_{k1}} Y_{k1} \geq y \right) \right] + \lambda_0 P\left(\sum_{i=1}^{\eta_{01}} Y_{01} \geq \frac{y}{2} \right) dy} = \frac{1}{c-\mu}.$$

ix) For this model, the small claim condition means that there exists the Cramer-Lundberg exponent ε which is the smallest possible solution of the equation

$$\lambda_1[1-g_1(Ee^{\varepsilon Y_{11}})]+\lambda_0[1-g_0(Ee^{2\varepsilon Y_{01}})]+\lambda_2[1-g_2(Ee^{\varepsilon Y_{21}})]=c\varepsilon.$$

In that case $\psi(u) \le e^{-\varepsilon u}$ and choosing $\alpha \in (0,1)$ we can find appropriate premium income rate c, in such a way that $\psi(u) \le \alpha$.

If additionally

$$\sum_{k=1}^{2}\left\{\frac{\lambda_k}{\mu}\int_0^\infty xe^{\varepsilon x}P\left(\sum_{i=1}^{\eta_{k1}}Y_{k1}\ge x\right)dx\right\}+\frac{\lambda_0}{\mu}\int_0^\infty xe^{\varepsilon x}P\left(\sum_{i=1}^{\eta_{01}}Y_{01}\ge\frac{x}{2}\right)dx<\infty,$$

then the following Cramer-Lundberg approximation of the probability of ruin holds

$$\lim_{u\to\infty}e^{\varepsilon u}\psi(u)=\frac{c-\mu}{\varepsilon\left\{\sum_{k=1}^{2}[\lambda_k\int_0^\infty xe^{\varepsilon x}P\left(\sum_{i=1}^{\eta_{k1}}Y_{k1}\ge x\right)dx]+\lambda_0\int_0^\infty xe^{\varepsilon x}P\left(\sum_{i=1}^{\eta_{01}}Y_{01}\ge\frac{x}{2}\right)dx\right\}}.$$

Scetch of the proof: The Total probability formula entails that the c.d.f. of Y_1 is

$$F_{Y_1}(x)=\frac{\lambda_1}{\lambda}P(\sum_{i=0}^{\eta_{11}}Y_{11}<x)+\frac{\lambda_2}{\lambda}P(\sum_{i=0}^{\eta_{21}}Y_{21}<x)+\frac{\lambda_0}{\lambda}P(\sum_{i=0}^{\eta_{01}}Y_{01}<\frac{x}{2}) \tag{13}$$

Using the double expectation formula we obtain their mean and LST

$$EY_1=\frac{\lambda_1}{\lambda}E(\eta_{1s})E(Y_{11})+\frac{2\lambda_0}{\lambda}E(\eta_{0s})E(Y_{01})+\frac{\lambda_2}{\lambda}E(\eta_{2s})E(Y_{21})=:\frac{\mu}{\lambda},$$

$$Ee^{-sY_1}=\frac{\lambda_1}{\lambda}g_1(Ee^{-sY_{11}})+\frac{\lambda_2}{\lambda}g_2(Ee^{-sY_{21}})+\frac{\lambda_0}{\lambda}g_0(Ee^{-2sY_{01}}).$$

In order to complete the proof we just replace these expressions in the well known formulae for the C-L model.

5 Conclusive remarks

Due to their feasibility risk models with dependent classes of business have recently attracted much attention is scientific literature. Although in general multivariate case with common shocks the formulae for their characteristics are huge, here using the bivariate model we show that these models can be reduced to the C-L case. This allows us to use already known results which reasonably simplifies the calculations. In analogous way we can reduce similar multivariate (with more then two classes of business) risk models with arbitrary common shocks to C-L model. Many particular cases can investigated. For example the numbers of claims in different groups can be of any discrete integer probability type. η_{01}, η_{11}, η_{21} can be constants, Poisson, Geometric, Negative binomial, Truncated geometric, Logarithmic, Binomial ets. We mention these, because there are explicit formulae and Panjer's type recursions (see [15]) for their compounds. These three r. vs. are not obligatory identically distributed. In case when the number of summands is governed by some distribution which has no investigated compound then higher-order asymptotic expansions presented e.g. in [1] and the references there in can be very useful. The experience of the author shows that the Monte-Carlo method is flexible, easy to implement, not so time consuming, and gives good results. See e.g. [7].

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