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MODEL OF LEARNING THROUGH PROBLEM SOLVING IN GEOGEBRA ACTIVITY ON THE TOPIC „SUM OF THE ANGLES IN A TRIANGLE“

DENITSA D. EFTIMOVA

ABSTRACT: *The paper proposes a model of e-interactive mathematics learning in the GeoGebra Activity/Classroom cloud platform. Methodological development is related to solving educational problems and arriving at an idea for solving the tasks, through research with dynamic constructions. The model aims to master the elements of the research approach.*

KEYWORDS: *learning problem, problem situation, hypothesis, angles in a triangle, GeoGebra*

2020 Math. Subject Classification: *97 Mathematics education*

1 Introduction

The best way to learn is to act - ask and then act. The best way to teach is to get students to ask and then act. Don't preach facts, stimulate action!

Paul Halmosh

Since the beginning of education, ways have been sought for its improvement and development. Researchers and educators make attempts to find those niches of education that show gaps in their content or the need to update previously applied methods or replace them with new ones desired by students, because in them lies a difficult problem, the analysis of which in time he sought his development. [7]

Increasing students' activity in the learning process and strengthening their motivation to learn is a major challenge facing the modern Bulgarian school. Educators' efforts are aimed at finding ways

to stimulate personal expression and bring in external motivation. With the advent of information technology and the development of learning environments, the possibilities for this are increasing. [4]

The development and implementation of innovative didactic concepts and pedagogical strategies based on the use of technology is a condition for improving the learning process in mathematics. The expectation is that they will provide a bridge for learning to move from school to beyond. [6]

Teaching mathematics in an electronic environment is an opportunity to use technologies for object visualization, research, experimentation, simulations, augmented reality, data processing, teamwork. The combination of appropriate electronic resources and environments in the traditional classroom is a prerequisite for an effective interactive mathematics lesson. [1]

The paper proposes a model of e-interactive mathematics learning through problem solving in the GeoGebra Activity/Classroom cloud platform. The methodological development is based on the topic of the educational content for 7th grade - "Sum of the angles in a triangle".

2 Exposure

In the proposed lesson development, a model for teaching mathematics to students in an electronic environment with solving learning challenges is presented. The technology used in the model is the cloud platform for dynamic mathematics GeoGebra. The main method used is learning by solving problems.

Problem-solving learning is an individual or group activity. It is based on setting a problem that students can face in their everyday life. They are expected to look at the problem, analyse it and explore possible solutions. All this is aimed at the students and contributes to increasing their motivation and activity in learning. [2]

Students are presented with problematic life situations in order to make them exert effort and enthusiasm in order to reach the desired result and solve the problem.

Solving a learning problem can be defined as an independent activity of the student to acquire new knowledge and skills, allowing the unknown elements of the problem to become known. [5]

In learning through problem solving, students are presented with a problem that they cannot solve with the knowledge they have, or they are placed in unfamiliar situations. Therefore, they look for ways to independently solve or with the help of the teacher analyse the problem, formulate a hypothesis, carry out research, observations, comparisons, analyse the results and prove the hypothesis. [3]

Lesson topic: Sum of angles in a triangle

Interactive sheet in *GeoGebra Activity*:
https://www.geogebra.org/m/njajaggf

Class: VII

2.1. Place of the lesson in the curriculum

Section: Basic geometric figures

Introduction to Geometry. Basic geometric shapes and constructions.

Adjacent angles, opposite angles. Perpendicular lines.

Corners obtained by the intersection of two straight lines with a third.

Sign of parallelism of two lines.

Axiom of parallel lines.

Properties of parallel lines.

A triangle.

Sum of the angles in a triangle.

An exterior angle of a triangle.

2.2. Concepts and skills:

✓ new concepts - none

✓ new skills – finding an unknown angle of a triangle by given two other angles, by a given ratio of the angles in the triangle,

by a given relationship for two of the angles in the triangle; is able to apply the relationships between the angles in a triangle;

✓ supporting concepts - lines, rays, angles, right angles, adjacent angles, triangles;

✓ support skills - determines the type of angles and finds angles.

2.3. Main learning objective of the lesson:

Formation of skills to apply the theorem of the sum of the angles in a triangle.

Global creative goal of the lesson:

Finding the relationship between the angles of a triangle.

2.4. Progress of the lesson

The lesson begins with an update of students' knowledge of the types of angles obtained when two parallel lines intersect with a third and their properties. The teacher has prepared an introductory GeoGebra task (applet1/Fig.1) https://www.geogebra.org/m/dytmjbpw), and students answer guided questions about it.

Problem 1. Name all the pairs of angles obtained by the intersection of the parallel lines **a** and **b** with the line **c**. Record their properties.

What types of angles are formed when two parallel lines intersect with a third?

Indicate them on the drawing and record their properties.

What other angles do you see in the drawing? Write them down.

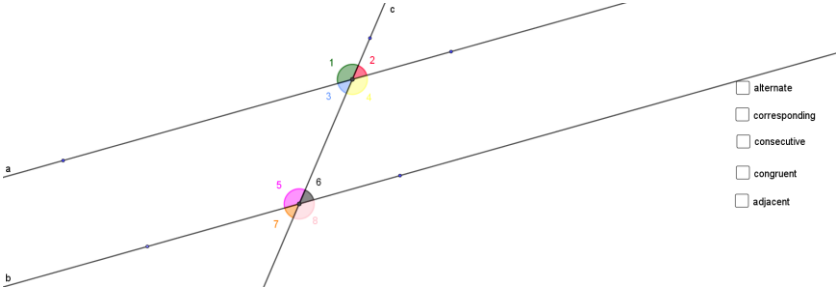


Fig. 1. Applet to problem 1

Students are reminded of the property of parallel lines, what is the sum of two adjacent angles, and the property of opposite (vertical) angles. Students find that when the angles are changed, by moving the lines defining them, the properties of the angles are preserved.

The elements of a triangle are recalled and the theme of the lesson - "Sum of the angles in a triangle" is motivated.

Creating a problem situation

Purpose: motivation to introduce the new knowledge through a practical life situation.

Study problem to discover new knowledge:

Problem 2. Imagine that your father has to replace the roof of the house and he needs to know what angle the slope will be, knowing that the angle that the supporting beam makes with the roof is 63 degrees. (Fig.2)

Attention! The main (bearing beam) is perpendicular to the foundation!

Fig. 2. Illustration of the learning problem from problem 2

Problem identification

• Analysis of the problem situation

What are the known elements?

A right triangle with an acute angle of $63°$.

What are the unknown elements?

The other sharp corner.

Is there a relationship between the given and sought items?

The given and sought elements are angles in a triangle.

• Awareness of the problem

If two of the angles in a triangle are known, can we find the third angle?

Students **formulate a problem**: To solve the problem, we need to find a relationship between the angles of a triangle.

Formulation of the hypothesis

Purpose: by comparing data to discover the common and different between the angles of different types of triangles and synthesis of the information to formulate a hypothesis.

Problem 3. Complete the table (Table 1) using the drawing (Fig.3) and make the necessary calculations.

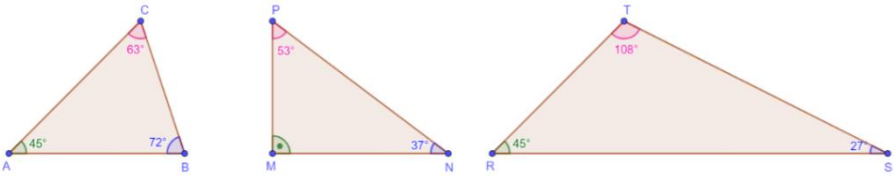


Fig. 3. Applet to problem 3

Table 1

	Type of triangle according to its angles	α	β	γ	$\alpha+\beta+\gamma$
ΔABC	acute	45°	72°	63°	180°
ΔMNP	right-angled	90°	37°	53°	180°
ΔRST	obtuse	45°	27°	108°	180°

Compare the results for the sum of the angles in each triangle. Write an equation that connects the angles in the triangle.
With this problem, students discover that in any triangle, the sum of the angles is 180°.

Hypothesis testing

Objective: to prove the validity of the hypothesis through dynamic drawing research.

Problem 4. *Investigate through the dynamic drawing from applet 2 (Fig.4) the truth of the hypothesis that* $\alpha+\beta+\gamma=180°$.

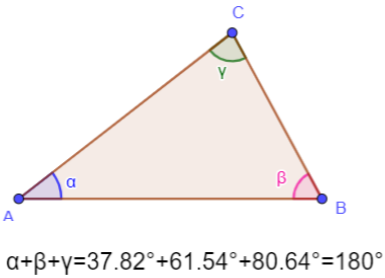


Fig.4. Applet to problem 4

Students change the size of the angles and the shape of the triangle by moving its vertices and see in the dynamic text that the sum is always $180°$.

If using an applet is not possible, the task can be done practically by having each student draw a random triangle, measure the angles with a protractor, and find their sum.

Generalization and introduction of new knowledge: concepts, theorems, properties, rules, etc.

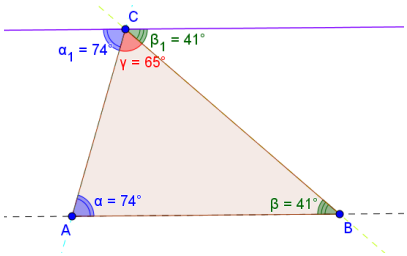
Objective: formulation of the theorem for the sum of the angles in a triangle and proof of the theorem.

*It is given **problem.5**.: Draw* ΔABC *and through the vertex C construct a line p parallel to the line AB.*

а) Explain why $\sphericalangle\alpha = \sphericalangle\alpha_1$ *and* $\sphericalangle\beta = \sphericalangle\beta_1$*;*

б) Does the sum depend on the measures of $\sphericalangle\alpha_1$ *and* $\sphericalangle\beta_1$*?*

The task is visualized with applet 3
https://www.geogebra.org/m/xssjpw5a.



What are the angles α *and* α_1,β *and* β_1*? Why are they equal?*

What angle do the angles make? α_1, γ *and* β_1*?*
Why is the sum of the angles in a triangle 180°?

Then the theorem is stated and proved (Fig.5)

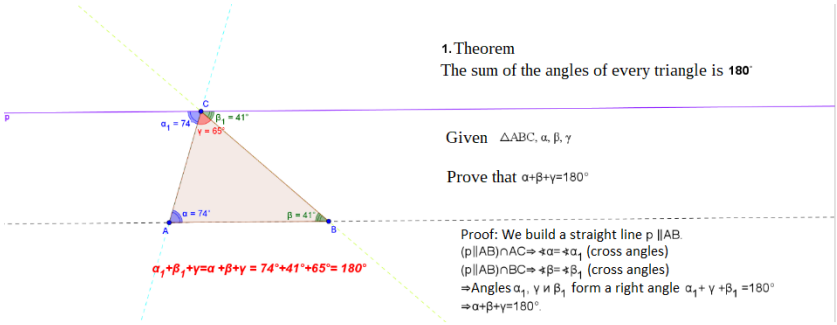


Fig.5 Applet to problem 5

In order to consolidate knowledge about the sum of the angles in a triangle, students also look at applet 4 (Fig. 6) https://www.geogebra.org/m/rge8ujwq, where the theorem is clearly presented.

Problem 6. Change the size of the angles of the triangle.

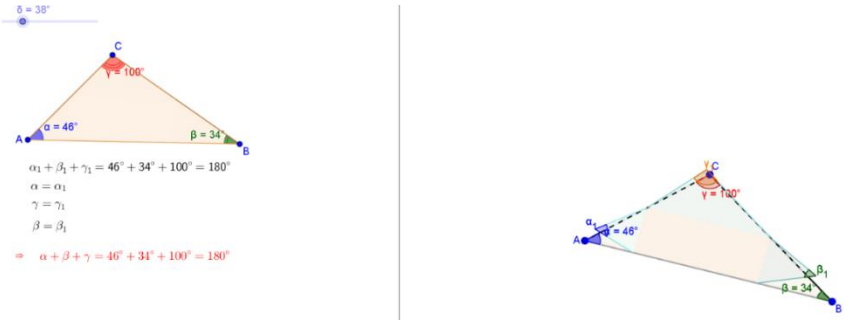


Fig.6 Applet to problem 6

Problem 7. *Answer the questions and justify the answers:*

- *Can a triangle have more than one right angle?*
- *Can a triangle have more than one obtuse angle?*
- *Can a triangle have a right angle and an obtuse angle?*

Using the answers and reasoning from behind.5. and through exploration and demonstration with a dynamic drawing from Applet 4, students arrive at the following conclusions:

1. There is no triangle that has more than one right angle;
2. $\Delta ABC, \measuredangle C = 90° \Rightarrow \alpha + \beta = 90°$
3. There is no triangle that has more than one obtuse angle;
4. There is no triangle that has both a right angle and an obtuse angle.

This task improves the ability to understand at a specific level the meaning of the logical conjunctions "and", "or", "if..., then..."; assessing the fidelity and rationality of a specific situation and justifying conclusions.

Resolving the problem situation

Objective: apply the theorem of the sum of the angles in a triangle to solve the problem situation.

After the theorem has already been formulated, the missing knowledge has been discovered, through which it is now possible to solve the problematic task.

Problem 8.

Given a rectangular ΔABC $\beta = 90°$ $\gamma = 63°$

$\alpha =?$

$\alpha + \beta + \gamma = 180°$

$\alpha + 90° + 63° = 180°$

$\alpha = 180° - (90° + 63°) = 27°$

The pitch of the roof is 27°.

Consolidation of new knowledge

Problem 9. *Which of the given angles can be angles in a triangle?*

- 60°; 50°; 70°;
- 66°; 44°; 80°;
- 54°; 45°; 80°.

Problem 10. *If* $\beta = 75°$, $\gamma = 50°$, *then* α *is*:

- 45°;
- 65°;
- 55°.

Problem 11. *B* ΔABC $\alpha = 60°, \beta = 30°$. *Triangle ABC is*:

- acute
- right-angled
- obtuse

Problem 12. A triangle can have:

- two right angles and one acute;

• two obtuse angles and one sharp;
• one straight and two acute angles.

At the end of the lesson, the teacher assigns the homework. One of the tasks is done on applet5 (Fig.7)

https://www.geogebra.org/m/fcdjmynx.

Problem 13. *Find X and Y based on the Fig.7.*

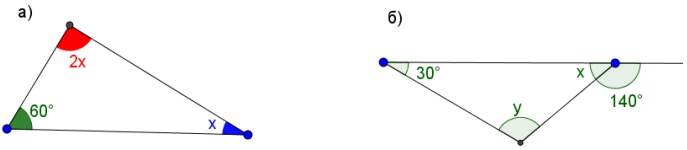


Fig. 7. Homework assignment with dynamic constructions

3 Conclusion

Modern technological possibilities allow the virtual class in mathematics to approach school learning in terms of effectiveness. For today's students, technology is part of their daily lives, and it's only natural that they use it while learning.

In conclusion, the following conclusions can be drawn:

• There is an interest on the part of the students in the learning content in mathematics in the section "Basic geometric figures", 7th grade;

• The use of ICT provides opportunities to increase the quality and efficiency of the educational process;

• Using lessons with dynamic GeoGebra software puts students in the position of discoverers, explorers and increases their motivation to learn.

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Деница Д. Ефтимова
"Stefan Karadzha" Secondary School, Kavarna
e-mail: deni_1258@abv.bg