IMPROVED LOCAL CONVERGENCE ANALYSIS OF THE INVERSE WEIERSTRASS METHOD FOR SIMULTANEOUS APPROXIMATION OF POLYNOMIAL ZEROS*

GYURHAN H. NEDZHIBOV

ABSTRACT: *In this work we establish new local convergence result with a-priori and a-posteriori error estimates for the Inverse Weierstrass iterative method for simultaneous approximations of polynomial zeros. Our approach enlarges the convergence radius and improves the known local convergence results.*

KEYWORDS: *Polynomial zeros, Simultaneous method, Weierstrass method, Durand-Kerner method, Inverse Weierstrass method, Local convergence*

1 Introduction

Let $P(z)$ be a monic polynomial

(1) $$P(z) = a_0 + a_1 z + \ldots + a_{n-1} z^{n-1} + z^n \,,$$

of degree $n \geq 2$, with simple real or complex zeros $\alpha_1, \alpha_2, \ldots, \alpha_n$, and let $z_1^{(0)}, z_2^{(0)}, \ldots, z_n^{(0)}$ be distinct reasonable close approximations of these zeros.

In this study we consider a simultaneous iterative method defined by

(2) $$\mathbf{z}^{(k+1)} = \mathbf{G}\left(\mathbf{z}^{(k)}\right) = \mathbf{G}^{k+1}\left(\mathbf{z}^{(0)}\right), \ \ k = 0, 1, 2, \ldots ,$$

where $\mathbf{G} : \mathbf{C}^n \to \mathbf{C}^n$ is a vector valued function with components

(3) $$G_i = G_i(\mathbf{z}) = \frac{z_i^2}{z_i + W_i(\mathbf{z})} \,, \ \mathbf{z} = (z_1, \ldots, z_n) \,, \ i = 1, \ldots, n \,,$$

and the *Weierstrass' correction* $W_i : \mathscr{D} \subset \mathbf{C}^n \to \mathbf{C}$ is defined by

(4) $$W_i(\mathbf{z}) = \frac{P(z_i)}{\prod_{j\neq i}^n (z_i - z_j)} \,, \ \ (i = 1, \ldots, n)$$

where $\mathscr{D}$ is the set of all vectors in $\mathbf{C}^n$ with distinct components. Then we can define the operator $W : \mathscr{D} \subset \mathbf{C}^n \to \mathbf{C}^n$ by $W(z) = (W_1(z), \ldots, W_n(z))$.

The method (2)-(3) was firstly introduced in [1], and some recent results were obtained in [2, 3, 4, 5]. The obtained results in these works are modifications of local convergence results of classical Weierstrass iterative method presented in [6, 7, 8, 9, 10, 11, 12].

Throughout this paper, we will use the p-norm defined by

(5) $$\|x\|_p = \left(\sum_{i=1}^n |x_i|^p \right)^{\frac{1}{p}}$$

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for some $1 \leq p \leq \infty$ and we will follow the usual convention that a summation over the empty set of indices equals 0, while a product over the same set equals 1. We use the function $d : \mathbf{C}^n \to \mathbf{R}_+$ defined by

$$d(x) = \min\{\delta(x), \gamma(x)\}, \tag{6}$$

where

$$\delta(x) = min_{i \neq j}|x_i - x_j| \text{ and } \gamma(x) = min_j|x_j| \quad (j = 1, \ldots, n). \tag{7}$$

Further, for a number p such that $1 \leq p \leq \infty$ we denote by q the conjugate exponent of p, i.e. q is defined by means of

$$\frac{1}{p} + \frac{1}{q} = 1 \text{ and } 1 \leq q \leq \infty.$$

We study the local convergence of the *Inverse Weierstrass method* (2)-(3) with respect to the function of initial conditions $E : \mathscr{C}^n \to \mathscr{R}_+$ defined as follows

$$E(z) = \frac{\|z - \alpha\|_p}{d(\alpha)}. \tag{8}$$

In our previous work (see [4])we have proved the following convergence result.

Theorem 1.1. *Let $P \in \mathscr{C}[z]$ be a monic polynomial of degree $n \geq 2$, where $\alpha = \{\alpha \in \mathscr{C}^n : \alpha_i \neq 0$ and $\alpha_i \neq \alpha_j$ for $i, j = 1, \ldots, n\}$ is the root vector of P, and let $1 \leq p \leq \infty$, $d = d(\alpha) = \min\{\delta, \gamma\}$, where $\delta = min_{j \neq i}|\alpha_i - \alpha_j|$ and $\gamma = min_i|\alpha_i|$ for $i, j = 1, \ldots, n$ ($i \neq j$). Suppose $z^{(0)} \in \mathscr{C}^n$ is an initial guess satisfying*

$$E(z^{(0)}) = \left\| \frac{z^{(0)} - \alpha}{d(\alpha)} \right\|_p < R(n, p) = \frac{\theta^{\frac{1}{n-1}} - 1}{2^{\frac{1}{q}}(\theta^{\frac{1}{n-1}} - 1) + (n-1)^{-\frac{1}{p}}}, \tag{9}$$

where

$$\theta = \frac{3b - 2 + \sqrt{b^2 - 4b + 20}}{2(b+1)} \quad \text{and} \quad b = 2^{\frac{1}{q}}. \tag{10}$$

Then the following statements hold true.
(i) CONVERGENCE. *The Inverse Weierstrass iteration (2)-(3) is well defined and converges quadratically to the root-vector α of P.*
(ii) A POSTERIORI ERROR ESTIMATE. *For all $k \geq 0$ we have the estimate*

$$\|z^{(k+1)} - \alpha\|_p \leq \lambda^{2^k} \|z^{(k)} - \alpha\|_p, \tag{11}$$

(iii) A PRIORI ERROR ESTIMATE. *For all $k \geq 1$ we have the estimate*

$$\|z^{(k)} - \alpha\|_p \leq \lambda^{2^k - 1} \|z^{(0)} - \alpha\|_p, \tag{12}$$

where $\lambda = E(z^{(0)})/R(n, p)$.

The main purpose of this work is to improve this theorem and obtain new local convergence result with larger radius of convergence.

2 Main Results

First, we introduce some auxiliary results. We will state following three known lemmas that we will use without proofs (the proofs may be found, e.g. in [12]).

Lemma 2.1. *Let $u \in \mathscr{C}^n$ and $1 \le p \le \infty$. Then*

$$|u_i| + |u_j| \le 2^{\frac{1}{q}} \|u\|_p \text{ for any } i, j = 1, 2, \ldots, n. \tag{13}$$

Lemma 2.2. *Let $u \in \mathscr{C}^n$ and $1 \le p \le \infty$. Then*

$$\left(\prod_{i=1}^{n} (1 + |u_i|) - 1 \right) \le \left(1 + \frac{\|u\|_p}{n^{1/p}} \right)^n - 1\,. \tag{14}$$

Lemma 2.3. *Let $n \in \mathscr{N}$, $t \ge 0$ and $0 \le \varphi \le 1$. Then*

$$(1 + \varphi t)^n - 1 \le \varphi \left((1+t)^n - 1 \right)\,. \tag{15}$$

Now, we will prove the following two lemmas.

Lemma 2.4. *Let $0 < t < 1/2^{1/q}$, $n \ge 2$ and*

$$\phi(t) = 1 + \frac{t}{(n-1)^{1/p}(1 - 2^{1/q} t)}\,. \tag{16}$$

Suppose that

$$\phi(t)^{n+1} < 2^{1/q}\,, \tag{17}$$

then it follows

$$\frac{1+t}{1-t} \phi(t)^{n-1} < 2\,. \tag{18}$$

Proof. It is easy to prove that

$$\tilde{t} = \frac{2^{\frac{1}{q(n+1)}} - 1}{2^{\frac{1}{q}} \left(2^{\frac{1}{q(n+1)}} - 1 \right) + (n-1)^{-\frac{1}{p}}} \tag{19}$$

is the unique root of the equation

$$\phi(t)^{n+1} = 2^{1/q}$$

in the interval $(0, 1/2^{1/q})$. Taking into account that the function $\phi(t)^{n+1}$ is continuous and strictly increasing on the interval $I = [0, \tilde{t}]$, we deduce that (17) is equivalent to

$$t < \tilde{t}\,.$$

From this and (19) it follows that

$$\frac{1+t}{1-t} < \frac{(2^{\frac{1}{q}} + 1)(2^{\frac{1}{q(n+1)}} - 1) + (n-1)^{-1/p}}{(2^{\frac{1}{q}} - 1)(2^{\frac{1}{q(n+1)}} - 1) + (n-1)^{-1/p}} \le \frac{3.2^{\frac{1}{n+1}} - 2}{2^{\frac{1}{n+1}}}\,. \tag{20}$$

From (17) it follows that

(21) $$\phi(t)^{n-1} < 2^{\frac{n-1}{q(n+1)}}.$$

The last two relations (20) and (21) it follows that

(22) $$\frac{1+t}{1-t}\,\phi(t)^{n-1} < \frac{3.2^{\frac{1}{n+1}}-2}{2^{\frac{1}{n+1}}}2^{\frac{n-1}{q(n+1)}} = 3.2^{\frac{n-1}{n+1}} - 2.2^{\frac{n-2}{n+1}} = \theta(n).$$

The function $\theta(n)$ is is continuous and strictly increasing for $n \geq 2$ and

(23) $$\lim_{n\to\infty}\theta(n) = 2.$$

Now from (22) and (23) follows the statement (18). The lemma is proved.

The next lemma is due to our previous work [4] in the case when $\left\|\frac{z^{(k)}-\alpha}{d(\alpha)}\right\|_p$ is replaced by $\frac{\|z^{(k)}-\alpha\|_p}{d(\alpha)}$.

Lemma 2.5. *Let $P \in \mathscr{C}[z]$ be a monic polynomial of degree $n \geq 2$, where $\alpha = \{\alpha \in \mathscr{C}^n : \alpha_i \neq 0$ and $\alpha_i \neq \alpha_j$ for $i,j = 1,\dots,n\}$ is the root-vector of P, $1 \leq p \leq \infty$. Let for any $k \geq 0$*

(24) $$E_k = E(z^{(k)}) = \frac{\|z^{(k)}-\alpha\|_p}{d} < \frac{1}{2^{\frac{1}{q}}},$$

where $d = d(\alpha)$ is defined by (6).Then the iteration $z^{(k)}$ is well defined and it has distinct components. Besides,

(25) $$\|z^{(k+1)}-\alpha\|_p \leq \sigma_k \|z^{(k)}-\alpha\|_p$$

and

(26) $$E(z^{(k+1)}) \leq \sigma_k E(z^{(k)}),$$

where

(27) $$\sigma_k = \sigma(E_k) = \frac{\frac{1}{1-E_k}\left(1+\frac{E_k}{(n-1)^{1/p}(1-2^{1/q}E_k)}\right)^{n-1} - 1}{1-\frac{E_k}{1-E_k}\left(1+\frac{E_k}{(n-1)^{1/p}(1-2^{1/q}E_k)}\right)^{n-1}}.$$

Proof. Using the triangle inequality, Lemma 2.1 and (24), we get for $i \neq j$

(28) $$\begin{aligned} |z_i^{(k)} - z_j^{(k)}| &\geq |\alpha_i-\alpha_j| - |z_i^{(k)}-\alpha_i| - |z_j^{(k)}-\alpha_j| \\ &= \left(1-\left|\frac{z_i^{(k)}-\alpha_i}{\alpha_i-\alpha_j}\right| - \left|\frac{z_j^{(k)}-\alpha_j}{\alpha_i-\alpha_j}\right|\right)|\alpha_i-\alpha_j| \\ &\geq \left(1-2^{\frac{1}{q}}\frac{\|z^{(k)}-\alpha\|_p}{d(\alpha)}\right)d(\alpha) = (1-2^{\frac{1}{q}}E_k)d > 0, \end{aligned}$$

which means that $z^{(k)}$ has distinct components, i.e. the iteration is well defined. Now we will prove the following estimates

$$|z_i^{(k+1)} - \alpha_i| \le \sigma_k |z_i^{(k)} - \alpha_i| \quad \text{for} \quad i = 1, 2, \ldots, n. \tag{29}$$

For easy of later comparisons, we will use the following equivalent form of (3)

$$z_i^{(k+1)} = z_i^{(k)} - \frac{W_i(z^{(k)})}{1 + \frac{W_i(z^{(k)})}{z_i^{(k)}}}, \quad i = 1, 2, \ldots, n, \tag{30}$$

which implies

$$z_i^{(k+1)} - \alpha_i = z_i^{(k)} - \alpha_i - \frac{W_i(z^{(k)})}{1 + \frac{W_i(z^{(k)})}{z_i^{(k)}}} = (z_i^{(k)} - \alpha_i)\left[1 - \frac{\prod_{j\neq i}^{n} \frac{z_i^{(k)} - \alpha_j}{z_i^{(k)} - z_j^{(k)}}}{1 + \frac{W_i(z^{(k)})}{z_i^{(k)}}}\right]$$

and consequently

$$(z_i^{(k+1)} - \alpha_i) = (z_i^{(k)} - \alpha_i)\left[\frac{1 - \prod_{j\neq i}^{n} \frac{z_i^{(k)} - \alpha_j}{z_i^{(k)} - z_j^{(k)}} + \frac{W_i(z^{(k)})}{z_i^{(k)}}}{1 + \frac{W_i(z^{(k)})}{z_i^{(k)}}}\right].$$

Therefore

$$|z_i^{(k+1)} - \alpha_i| = A_i^{(k)} |z_i^{(k)} - \alpha_i|, \tag{31}$$

where

$$A_i^{(k)} := \left|\frac{1 - \prod_{j\neq i}^{n} \frac{z_i^{(k)} - \alpha_j}{z_i^{(k)} - z_j^{(k)}} + \frac{W_i(z^{(k)})}{z_i^{(k)}}}{1 + \frac{W_i(z^{(k)})}{z_i^{(k)}}}\right|.$$

Now, we can bound the amplification factor $A_i^{(k)}$ as follows

$$A_i^{(k)} \le \frac{\left|\prod_{j\neq i}^{n} \frac{z_i^{(k)} - \alpha_j}{z_i^{(k)} - z_j^{(k)}} - 1\right| + \left|\frac{z_i^{(k)} - \alpha_i}{z_i^{(k)}}\right| \left|\prod_{j\neq i}^{n} \frac{z_i^{(k)} - \alpha_j}{z_i^{(k)} - z_j^{(k)}}\right|}{1 - \left|\frac{z_i^{(k)} - \alpha_i}{z_i^{(k)}}\right| \left|\prod_{j\neq i}^{n} \frac{z_i^{(k)} - \alpha_j}{z_i^{(k)} - z_j^{(k)}}\right|}. \tag{32}$$

We next establish the following inequalities

$$|z_i^{(k)}| \ge |\alpha_i| - |z_i^{(k)} - \alpha_i| \ge d - \|z^{(k)} - \alpha\|_p \ge (1 - E_k)d > 0. \tag{33}$$

It follows from (28) and the definition of $E(z^{(k)})$ that

$$\left|\frac{z_i^{(k)} - \alpha_j}{z_i^{(k)} - z_j^{(k)}}\right| = \left|1 + \frac{z_j^{(k)} - \alpha_j}{z_i^{(k)} - z_j^{(k)}}\right| \le 1 + \frac{|z_j^{(k)} - \alpha_j|}{(1 - 2^{\frac{1}{q}} E_k) d(\alpha)}. \tag{34}$$

From (32), the last two inequalities (34) and (33), and Lemma 2.2, we obtain

$$A_i^{(k)} \leq \frac{\left(1+\frac{E_k}{(n-1)^{\frac{1}{p}}(1-2^{\frac{1}{q}}E_k)}\right)^{n-1} - 1 + \frac{E_k}{1-E_k}\left(1+\frac{E_k}{(n-1)^{\frac{1}{p}}(1-2^{\frac{1}{q}}E_k)}\right)^{n-1}}{1-\frac{E_k}{1-E_k}\left(1+\frac{E_k}{(n-1)^{\frac{1}{p}}(1-2^{\frac{1}{q}}E_k)}\right)^{n-1}} \tag{35}$$

and consequently

$$A_i^{(k)} \leq \frac{\frac{1}{1-E_k}\left(1+\frac{E_k}{(n-1)^{\frac{1}{p}}(1-2^{\frac{1}{q}}E_k)}\right)^{n-1} - 1}{1-\frac{E_k}{1-E_k}\left(1+\frac{E_k}{(n-1)^{\frac{1}{p}}(1-2^{\frac{1}{q}}E_k)}\right)^{n-1}} = \sigma_k\,.$$

Finally, from the last expression and (31) we obtain (29). Taking the p-norm in (29), we deduce the inequality in (25). We get the second inequality in (26) taking the p-norm and by dividing both sides of inequality (29) by $d(\alpha)$.

Now we are ready to state the main result of this paper which improves the previous results introduced in [4].

Theorem 2.6. *Let $P \in \mathscr{C}[z]$ be a monic polynomial of degree $n \geq 2$, where $\alpha = \{\alpha \in \mathscr{C}^n : \alpha_i \neq 0$ and $\alpha_i \neq \alpha_j$ for $i, j = 1, \ldots, n\}$ is the root vector of P, and let $1 \leq p \leq \infty$. Suppose $z^{(0)} \in \mathscr{C}^n$ is an initial guess satisfying*

$$E(z^{(0)}) = \frac{\|z^{(0)} - \alpha\|_p}{d(\alpha)} < R(n,p) = \frac{2^{\frac{1}{q(n+1)}} - 1}{2^{\frac{1}{q}}(2^{\frac{1}{q(n+1)}} - 1) + (n-1)^{-\frac{1}{p}}}\,. \tag{36}$$

Then the following statements hold true.
(i) CONVERGENCE. *The Inverse Weierstrass iteration (2)-(3) is well defined and converges quadratically to the root-vector α of P.*
(ii) A POSTERIORI ERROR ESTIMATE. *For all $k \geq 0$ we have the estimate*

$$\|z^{(k+1)} - \alpha\|_p \leq \lambda^{2^k}\|z^{(k)} - \alpha\|_p\,, \tag{37}$$

(iii) A PRIORI ERROR ESTIMATE. *For all $k \geq 1$ we have the estimate*

$$\|z^{(k)} - \alpha\|_p \leq \lambda^{2^k - 1}\|z^{(0)} - \alpha\|_p\,, \tag{38}$$

where $\lambda = E(z^{(0)})/R(n,p)$.

Proof. (i) From the Lemma 2.4 it follows that $R = R(n,p)$ is the unique solution of the equation $\phi(t)^{n+1} = 2^{1/q}$ in the interval $(0, 1/2^{1/q})$, where $\phi(t)$ is defined by (16). First, we will prove that for any $k \geq 0$ the iteration $z^{(k)}$ in (2)-(3) is well defined and

$$E(z^{(k)}) \leq R\lambda^{2^k}\,.$$

We shall use mathematical induction to prove the statement. First, we confirm that the base case $k = 0$ is true due to definition of λ. Using the assumption $n \geq 2$ and that $(n-1)^{-\frac{1}{p}} > 0$ for any

$1 \le p < \infty$, it can be shown that $R < 1/2^{1/q}$. Then the initial assumption $E(z^{(0)}) < R$ implies $E(z^{(0)}) < 1/2^{1/q}$. From this and Lemma 2.5 we deduce that the iteration $z^{(0)}$ is well defined.

Now, we will prove that

$$\sigma(E_0) \le \lambda\,.$$

From (27) it follows that $\sigma(E_k)$ can be written in the following equivalent form

$$\sigma(E_k) = \frac{\left(1+\frac{E_k}{(n-1)^{1/p}(1-2^{1/q}E_k)}\right)^{n-1} - 1 + E_k}{1 - E_k - E_k\left(1+\frac{E_k}{(n-1)^{1/p}(1-2^{1/q}E_k)}\right)^{n-1}}\,, \tag{39}$$

which implies

$$\sigma(E_0) \le \frac{\left(\left(1+\frac{\lambda R}{(n-1)^{1/p}(1-2^{1/q}R)}\right)^{n-1} - 1\right) + \lambda R}{1 - R - R\left(1+\frac{R}{(n-1)^{1/p}(1-2^{1/q}R)}\right)^{n-1}}\,.$$

From Lemma 2.3, where $\varphi = \lambda$ and $t = \frac{R}{(n-1)^{1/p}(1-2^{1/q}R)}$ we deduce

$$\sigma(E_0) \le \frac{\lambda\left(\left(1+\frac{R}{(n-1)^{1/p}(1-2^{1/q}\lambda R)}\right)^{n-1} - 1\right) + \lambda R}{1 - R - R\left(1+\frac{R}{(n-1)^{1/p}(1-2^{1/q}R)}\right)^{n-1}} \le \lambda\sigma(R)\,, \tag{40}$$

where

$$\sigma(R) = \frac{\left(\left(1+\frac{R}{(n-1)^{1/p}(1-2^{1/q}R)}\right)^{n-1} - 1\right) + R}{1 - R - R\left(1+\frac{R}{(n-1)^{1/p}(1-2^{1/q}R)}\right)^{n-1}}\,.$$

It is easy to show that the assumption $\sigma(R) < 1$ is equivalent to the assumption defined by (18), where $t = R$. Therefore, from (40) using Lemma 2.3 and the definition of R we deduce that $\sigma(E_0) \le \lambda$.

Suppose that for any $k \ge 0$ is fulfilled

$$E(z^{(k)}) \le R\lambda^{2^k} \tag{41}$$

and we will prove that

$$E(z^{(k+1)}) \le R\lambda^{2^{k+1}}\,.$$

From (39) and the assumption by induction we obtain

$$\sigma(E_k) \le \frac{\left(\left(1+\frac{\lambda^{2^k}R}{(n-1)^{1/p}(1-2^{1/q}R)}\right)^{n-1} - 1\right) + \lambda^{2^k}R}{1 - R - R\left(1+\frac{R}{(n-1)^{1/p}(1-2^{1/q}R)}\right)^{n-1}} \le \lambda^{2^k}\sigma(R) < \lambda^{2^k}\,. \tag{42}$$

Therefore, from (42), the assumption (41) and the estimate (26) it follows that

$$E(z^{(k+1)}) \le \sigma_k E(z^{(k)}) \le \lambda^{2^k} R\lambda^{2^k} = \lambda^{2^{k+1}}R\,.$$

Using the inequality $R < 1/2^{1/q}$ it follows that $E(z^{(k+1)}) < 1/2^{1/q}$, which implies by Lemma 2.5 that the iteration $z^{(k+1)}$ is well defined.

(ii) From (42) and Lemma 2.5 estimate (25) it is trivial to prove that

$$\|z^{(k+1)} - \alpha\|_p \leq \lambda^{2^k} \|z^{(k)} - \alpha\|_p .$$

(iii) From the assertion (ii) and the sum of geometric progression, we obtain

$$\begin{aligned} \|z^{(k)} - \alpha\|_p &\leq \lambda^{2^{k-1}} \|z^{(k-1)} - \alpha\|_p \leq \lambda^{2^{k-1}} \lambda^{2^{k-2}} \|z^{(k-2)} - \alpha\|_p \leq \dots \\ &\leq \lambda^{2^{k-1}} \lambda^{2^{k-2}} \dots \lambda^{2^0} \|z^{(0)} - \alpha\|_p = \lambda^{2^k - 1} \|z^{(0)} - \alpha\|_p . \end{aligned}$$

which implies (38). The theorem is proved.

In the case of use the maximum vector norm, i.e. if $p = \infty$, we obtain the following corollary of Theorem 2.6.

Theorem 2.7. *Let $P \in \mathscr{C}[z]$ be a monic polynomial of degree $n \geq 2$, where $\alpha = \{\alpha \in \mathscr{C}^n : \alpha_i \neq 0$ and $\alpha_i \neq \alpha_j$ for $i, j = 1, \dots, n\}$ is the root vector of P, and let $p = \infty$. Suppose $z^{(0)} \in \mathscr{C}^n$ is an initial guess satisfying*

$$E(z^{(0)}) = \frac{\|z^{(0)} - \alpha\|_\infty}{d(\alpha)} < \frac{2^{\frac{1}{n+1}} - 1}{2.2^{\frac{1}{n+1}} - 1} . \tag{43}$$

Then the Inverse Weierstrass iteration (2)-(3) is well defined, converges quadratically to the root-vector α of P and the error estimates (37) and (38) are hold true.

3 Conclusion

In this work we investigate convergence analysis of the Inverse Weierstrass iterative method for simultaneous approximation of polynomial zeros. Our goal was to obtain new local convergence analysis results and to improve the existing ones. We establish a new convergence theorem with larger radius of convergence.

REFERENCES:

[1] Nedzhibov, G.H., Similarity transformations between some companion matrices, *AIP Conf. Proc.* **1631**, (2014), 375–382.

[2] Nedzhibov, G.H., Inverse Weierstrass-Durand-Kerner Iterative Method, *International Journal of Applied Mathematics, ISSN:2051-5227*, **28**, Issue.2, (2013), 1258–1264.

[3] Nedzhibov, G.H., On local convergence analysys of the Inverse WDK method, MATHTECH 2016, *Proceedings of the international conference***1**, (2016), 118–126.

[4] Nedzhibov, G.H., Local convergence of the Inverse Weierstrass method for simultaneous approximation of polynomial zeros, *International Journal of Mathematical Analysis* **10**, No. 26, (2016), 1295–1304.

[5] Nedzhibov, G.H., Convergence of the modified inverse Weierstrass method for simultaneous approximation of polynomial zeros, *Communications in Numerical Analysis* **No. 1**, (2016), 74–80.

[6] Kjurkchiev, N.V., Markov, S.M., Two interval methods for algebraic equations with real roots, *Pliska Stud. Math. Bulg.* **5**, (1983), 118–131.

[7] Zheng, S.M., On convergence of the Durand-Kerners method for finding all roots of a polynomial simultaneously, *Kexue Tongbao* **27**, **1982**, 1262–1265.

[8] Petković, M., On initial conditions for the convergence of simultaneous root-finding methods, *Computing* **57**, (1996), 163–177.

[9] Han, D.F., The convergence of the Durand-Kerner method for simultaneously finding all zeros of a polynomial, *J. Comput. Math.* **18**, (2000), 567–570.

[10] Niel, A.M., The simultaneous approximation of polynomial roots, *Computers and Mathematics with applications*, **41**, (2001), 1–14.

[11] Proinov, P.D., A new semilocal convergence theorem for the Weierstrass method from data at one point, *C. R. Acad. Bulg. Sci.* **59**, No 2, (2006), 131–136.

[12] Proinov, P.D., Petkova, M.D., A new semilocal convergence theorem for the Weierstrass method for finding zeros of a polynomial simultaneously, *Journal of complexity*, **30**, Issue.3, (2014), 366-380.

Gyurhan Nedzhibov
Konstantin Preslavsky University of Shumen
E-mail: gyurhan@shu.bg