Годишник на ШУ „Епископ К. Преславски“
Факултет по математика и информатика, том XVIII C, 2017, стр. 3- 14

A TRIANGULAR MODEL OF A COUPLING OF DISSIPATIVE AND ANTIDISSIPATIVE OPERATORS WITH REAL SPECTRA*

Galina S. Borisova

ABSTRACT: *This paper finishes the description of the large class of nonselfadjoint operators with real spectra, presented as a coupling of dissipative and antidissipative operators and having a limit of the corresponding correlation functions.*

KEYWORDS: *Nonselfadjoint operator, dissipative operator, triangular model, characteristic operator function, operator colligation, coupling*

1 Introduction

This paper is dedicated to the further development of one of the directions in the nonselfadjoint operator theory founded by M.S. Livšic and his associates.

After the classical book of John von Neumann "The Mathematical Foundations of Quantum Mechanics" (where the quantum mechanics is presented as a unified theory based on the spectral theory of selfadjoint operators in Hilbert spaces) the efforts of many mathematicians show that the spectral analysis of nonselfadjoint operators cannot be made to fit into the framework of the theory of selfadjoint operators and its simplest generalizations. Nonselfadjoint operators arise in the discussion of processes that proceed without conservation of energy.

In the end of 1970s there were many prerequisites for investigations of nonselfadjoint operators. The theory of nonselfadjoint operators is based on:

— the theory of the characteristic functions and the triangular models of M.S. Livšic;

*Partially supported by Scientific Research Grant DDVU 02/91 of MON and Scientific Research Grant RD-08-120/2017 of Shumen University.

— the theory of dilations and functional models of B.Sz.-Nagy and C. Foias;

— the scattering theory of P. Lax and R. Phillips.

The first major method in the nonselfadjoint operator theory was initiated in the middle of the 1940s by M.S. Livšic [11, 13]. The main tool here is the characteristic operator function, which is associated with an arbitrary operator A in a Hilbert space H in the following way

$$I - i\Phi(A - \lambda I)^{-1}\Phi^* J,$$

where J is a signature operator (i.e. $J = J^* = J^{-1}$) and

$$\Phi^* J\Phi = \frac{A - A^*}{i}.$$

This function serves as a unitary invariant of the operator A and in many important cases it is much easier to analyze than the original operator. The characteristic operator function has intriguing properties. The main point is that there is a relation between invariant subspaces of the operator and factorizations of its characteristic function. This result is proved by M.S. Livšic and V.P. Potapov in [17]. This relation and Potapov's theorem allow to M.S. Livšic to construct the so-called triangular model of nonselfadjoint operators with finite dimensional imaginary parts ([12]).

Let H be a separable Hilbert space and let A be a bounded linear nonselfadjoint operator in H with a finite dimensional imaginary part $\dim(A - A^*)H < +\infty$ (i.e. the so-called operator with a finite non-hermitian rank.) Analogously it can be considered the case when the imaginary part of A belongs to the trace class.

In the case when the operator A is a dissipative operator in a Hilbert space H (i.e. $(A - A^*)/i \geq 0$) it follows immediately that there exists the limit

$$\lim_{t\to+\infty}(e^{itA}f, e^{itA}f), \quad f \in H.$$

The existence of this limits ensures the existence of the wave operator $W_+(A^*, A)$ as a weak limit

$$(W_+(A^*, A)f, g) = \lim_{t\to+\infty}(e^{-itA^*}e^{itA}f, g)$$

for the couple (A^*, A) $(f, g \in H)$, obtaining the correlation function $V(t, s) = (e^{itA}f, e^{isA}f)$ of the dissipative curves $e^{itA}f$ and the limit of the correlation function.

These results for the dissipative operators are presented in the works of M.S. Livšic and his associates (see, for example, [6, 16, 15, 14, 5]).

Naturally there arises the question: are there nondissipative operators A in a Hilbert space H for which there exist the limits

$$\lim_{t\to+\infty}(e^{itA}f, e^{itA}f), \quad \lim_{t\to-\infty}(e^{itA}f, e^{itA}f).$$

It turns out that there exists a larger class of bounded nondissipative operators in a Hilbert space which solve the question, mentioned above. This class of bounded linear nondissipative operators, presents couplings of dissipative and antidissipative operators with finite dimensional imaginary parts and real absolutely continuous spectra. (Analogously it can be considered the case when imaginary part of the operator belongs to the trace class.) The triangular model of the operators from this class is introduced by the author in [1] and investigated in [7, 2, 8]. The natural consideration of this class follows from the system-theoretic significance of the colligation which is connected with the multiplication theorem of the corresponding correlation function. In [7] the asymptotic behaviour of the nondissipative curves $e^{itA}f$ as $t \to \pm\infty$, $f \in H$, has been obtained, which ensures the existence and the explicite form of the limits $\lim\limits_{t\to\pm\infty}(e^{itA}f, e^{itA}f)$.

2 The triangular model

In the paper [1] the author has introduced the model which is a coupling of a dissipative and antidissipative operators with real spectra and finite dimensional imaginary parts. In this paper we will complete the description of a large class of nondissipative operators, present as a coupling of dissipative and antidissipative operators with real spectra and finite dimensional imaginary parts.

In [1] it has been introduced the model

$$\begin{aligned} Af(x) = \alpha(x)f(x) - i\int_0^x f(\xi)\Pi(\xi)S^*\Pi^*(x)d\xi + \\ + i\int_x^l f(\xi)\Pi(\xi)S\Pi^*(x)d\xi + i\int_0^x f(\xi)\Pi(\xi)L\Pi^*(x)d\xi, \end{aligned} \tag{1}$$

where the matrix function $\Pi(x)$ is a measurable $n \times m$ $(1 \leq n \leq m)$ matrix function on $[0, l]$, whose rows are linearly independent at each point of a set with a positive measure and satisfying the condition

$$tr\ \Pi^*(x)\Pi(x) = 1, \tag{2}$$

the function $\alpha(x)$ is a bounded nondecreasing function in $[0,l]$ which is continuous at 0 and continuous from the left in $(0,l]$, the matrix $L : \mathbb{C}^m \longrightarrow \mathbb{C}^m$ with $\det \neq 0$, $L^* = L$ has the form

$$L = J_1 - J_2 + S + S^*, \tag{3}$$

$$J_1 = \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix},\ J_2 = \begin{pmatrix} 0 & 0 \\ 0 & I_{m-r} \end{pmatrix},\ S = \begin{pmatrix} 0 & 0 \\ \widehat{S} & 0 \end{pmatrix},$$

I_k is the identity matrix in $\mathbb{C}^k$ $(k = r,\ m-r)$, $\widehat{S}$ is $(m-r)\times r$ matrix, r is the number of positive eigen values, $m-r$ is the number of negative eigen values of L, $\{f(x) = (f_1(x), f_2(x), \ldots, f_n(x)) : f_k(x) \in \mathbf{L}^2(0,l)\} = \mathbf{L}^2(0,l;\mathbf{C}^n)$ is a Hilbert space with a scalar product

$$(f(x), g(x)) = \int\limits_0^l f(x)g^*(x)dx, \qquad f(x), g(x) \in \mathbf{L}^2(0,l;\mathbf{C}^n).$$

It has to mention that every matrix L with $L = L^*$, $\det L \neq 0$, can be presented in the form

$$L = V(J_1 - J_2 + S + S^*)V^*, \tag{4}$$

where $V : \mathbb{C}^m \longrightarrow \mathbb{C}^m$ is invertible matrix. The representation (4) follows from the unitary equivelence of L and a diagonal matrix with eigen values of the matrix L and the generalized inertia law [6].

In the case when the matrix function satisfies the condition

$$\Pi^*(x)\Pi(x)J_1 = J_1\Pi^*(x)\Pi(x)$$

the operator (1) is a coupling of dissipative and antidissipative operators with real spectra and finite dimensional imaginary parts, i.e. the operator A can be presented in the form

$$A = P_1AP_1 + P_2AP_2 + P_1AP_2$$

where P_1, P_2 are ortogonal projectors in $\mathbf{L}^2(0,l;\mathbf{C}^n)$, P_1A is dissipative operator on the subspace $P_1\mathbf{L}^2(0,l;\mathbf{C}^n)$ (i.e. the imaginary part of

P_1AP_1 is a nonegative operator) and P_2A is antidissipative operator on $P_2\mathbf{L}^2(0,l;\mathbf{C}^n)$ (i.e. the imaginary part of P_1AP_1 is nonpositive operator). In other words, if $Q(x)$ is a measurable $m \times n$ matrix function in $[0,l]$, satisfying the condition $\Pi(x)Q(x) = I$ for almost all $x \in [0,l]$, the operators $P_1, P_2 : \mathbf{L}^2(0,l;\mathbf{C}^n) \longrightarrow \mathbf{L}^2(0,l;\mathbf{C}^n)$, defined by the equalities

$$P_1f(x) = f(x)\Pi(x)J_1Q(x), \qquad P_2f(x) = f(x)\Pi(x)J_2Q(x)$$

are orthoprojectors in $\mathbf{L}^2(0,l;\mathbf{C}^n)$, then the operator A has the representation

$$A = P_1AP_1 + P_2AP_2 + P_1AP_2.$$

In the last equality P_1A is dissipative operator on the subspace $P_1\mathbf{L}^2(0,l;\mathbf{C}^n)$ (i.e. the imaginary part of P_1AP_1 is a nonnegative operator) and P_2A is antidissipative operator on $P_2\mathbf{L}^2(0,l;\mathbf{C}^n)$ (i.e. the imaginary part of the operator P_1AP_1 is nonpositive), P_1AP_1 and P_2AP_2 have real spectra, determined by the values of the real function $\alpha(x)$.

It turns out that every nonselfadjoint operator T in a Hilbert space H which is a coupling of dissipative and antidissipative operators with real spectra and finite dimensional imaginary parts can be presented in the form (1).

Theorem 1. *Let the bounded nonselfadjoint operator T in a Hilbert space H with real spectrum and finite dimensional imaginary part is a coupling of dissipative and antidissipative operators with real spectra determined by a nondecreasing function. Then the operator T is unitery equivalent to the model (1).*

Proof. Let the bounded nonselfadjoint operator $T : H \longrightarrow H$ be a coupling of a dissipative operator and an antidissipative one with finite dimensional imaginary parts and real spectra, determined by the nondecreasing function $\alpha(x) : [0,l] \longrightarrow \mathbb{R}$. Then T has the representation from the form

$$T = P_1TP_1 + P_2TP_2 + P_1TP_2,$$

where P_1, P_2 are orthogonal projectors in H and $H = H_1 \oplus H_2$, $H_1 = P_1H$, $H_2 = P_2H$, H_1 is an invariant subspace according to T, P_1TP_1 is a dissipative operator and P_2TP_2 is an antidissipative operator. The operators P_1TP_1 и P_2TP_2 can be embedded in operator colligations X_1 и X_2.

Now we apply the theorem for unitary equivalence of an operator with finite dimensional imaginary part and the triangular model of M.S. Livšic ([6], [16]) onto the principal subspaces in the case of a dissipative operator and in the case of antidissipative operator with real spectra. From the relations

$$\dim \frac{P_1TP_1 - P_1T^*P_1}{i}H < \infty, \qquad \dim \frac{P_2TP_2 - P_2T^*P_2}{i}H < \infty,$$

$$\frac{P_1TP_1 - P_1T^*P_1}{i} \geq 0, \qquad \frac{P_2TP_2 - P_2T^*P_2}{i} \leq 0$$

it follows that the operator

$$\frac{P_1TP_1 - P_1T^*P_1}{i}$$

has s_1 nonnegative eigen values in the subspace

$$\frac{P_1TP_1 - P_1T^*P_1}{i}H$$

and the operator

$$\frac{P_2TP_2 - P_2T^*P_2}{i}$$

has s_2 nonpositive eigen values in the subspace

$$\frac{P_2TP_2 - P_2T^*P_2}{i}H.$$

Then there exist triangular models $\widetilde{T}_1$ and $\widetilde{T}_2$ which are embedded in the operator colligations from the form

$$\widetilde{X}_1 = (\widetilde{T}_1; \mathbf{L}^2(0,l;\mathbf{C}^{n_1}), \widetilde{\Phi}_1, \mathbf{C}^{r_1}; I_{r_1}),$$

$$\widetilde{X}_2 = (\widetilde{T}_2; \mathbf{L}^2(0,l;\mathbf{C}^{n_2}), \widetilde{\Phi}_2, \mathbf{C}^{r_2}; -I_{r_2})$$

such that the operators P_kTP_k и $\widetilde{T}_k$ are unitary equivalent onto the principal subspaces of the colligations X_k и $\widetilde{X}_k$ $(k = 1, 2)$. It has to mention that the positive numbers n_1, r_1, n_2, r_2 satisfy the inequalities

$$n_k \leq \dim E_k \leq s_k \leq r_k, \quad k = 1, 2,$$

where E_1 и E_2 are linear span of channels elements of the colligations X_1 и X_2 correspondingly. The operators $\widetilde{T}_1$ and $\widetilde{T}_2$ have the form

$$\widetilde{T}_1 g_1(x) = \alpha(x) g_1(x) + i \int\limits_0^x g_1(\xi) \widetilde{\Pi}_1(\xi) I_{r_1} \widetilde{\Pi}_1^*(x) d\xi,$$

$$\widetilde{T}_2 g_2(x) = \alpha(x) g_2(x) - i \int\limits_0^x g_2(\xi) \widetilde{\Pi}_2(\xi) I_{r_2} \widetilde{\Pi}_2^*(x) d\xi,$$

$g_k(x) \in \mathbf{L}^2(0, l; \mathbb{C}^{n_k})$ $(k = 1, 2)$. $\widetilde{\Pi}_k(\xi)$ are $n_k \times r_k$ $(k = 1, 2)$ matrices functions with linearly independent rows onto the subspaces with positive measure, the operators $\widetilde{\Phi}_k : \mathbf{L}^2(0, l; \mathbb{C}^{n_k}) \longrightarrow \mathbb{C}^{r_k}$ are defined by the equalities

$$\widetilde{\Phi}_k g_k(x) = \int\limits_0^l g_k(x) \widetilde{\Pi}_k(x) I_{r_k} dx,$$

where $g_k(x) \in \mathbf{L}^2(0, l; \mathbb{C}^{n_k})$, I_{r_k} is identity matrix in $\mathbb{C}^{r_k}$ $(k = 1, 2)$.

From the unitary equivalence there exist the unitary operators

$$U_k : \mathbf{L}^2(0, l; \mathbb{C}^{n_k}) \longrightarrow P_k H, \quad k = 1, 2,$$

such that

$$\widetilde{T}_k = U_k^* P_k T P_k U_k \tag{5}$$

onto the principal subspaces of $\widetilde{T}_k$ и $P_k T P_k$ $(k = 1, 2)$.

Let us consider now the space

$$\mathbf{L}^2(0, l; \mathbb{C}^n) = \mathbf{L}^2(0, l; \mathbb{C}^{n_1}) \oplus \mathbf{L}^2(0, l; \mathbb{C}^{n_2})$$

and the orthoprojectors in $\mathbf{L}^2(0, l; \mathbb{C}^n)$ defined by the equalities

$$\widetilde{P}_1 = \begin{pmatrix} I_{n_1} & 0 \\ 0 & 0 \end{pmatrix} \quad \widetilde{P}_2 = \begin{pmatrix} 0 & 0 \\ 0 & I_{n_2} \end{pmatrix}.$$

Let us choose the numbers m и r such that they satisfy the inequalities

$$n_1 \leq \dim E_1 \leq s_1 \leq r_1 \leq r,$$

$$n_2 \le \dim E_2 \le s_2 \le r_2 \le m - r.$$

Now we considr the matrix $L : \mathbb{C}^m \longrightarrow \mathbb{C}^m$ with $\det L \ne 0$, $L = L^*$ and with r positive and $m-r$ negative eigen values from the form

$$L = J_1 - J_2 + S + S^*,$$

where

$$J_1 = \begin{pmatrix} I_r & 0 \\ 0 & 0 \end{pmatrix}, \quad J_2 = \begin{pmatrix} 0 & 0 \\ 0 & I_{m-r} \end{pmatrix}, \quad S = \begin{pmatrix} 0 & 0 \\ \widehat{S} & 0 \end{pmatrix}.$$

Straightforward calculations show that if $f(x) \in \mathbf{L}^2(0,l;\mathbb{C}^n)$ and $f(x)$ is presented in the form $f(x) = (f_1(x), f_2(x))$, where $\widetilde{P}_k f(x) = f_k(x)$, $k = 1,2$, then

$$\begin{aligned} \widetilde{T}_1 f_1(x) &= \alpha(x) f_1(x) + i\int_0^x f_1(\xi)\widetilde{\Pi}_1(\xi) I_{r_1}\widetilde{\Pi}_1^*(x)d\xi = \\ &= \alpha(x)\widetilde{P}_1(f(x)) + i\int_0^x f(\xi)\widetilde{P}_1\widehat{\Pi}_1(\xi)L\widehat{\Pi}_1^*(x)\widetilde{P}_1 d\xi, \end{aligned} \tag{6}$$

$$\begin{aligned} \widetilde{T}_2 f_2(x) &= \alpha(x) f_2(x) - i\int_0^x f_2(\xi)\widetilde{\Pi}_2(\xi) I_{r_2}\widetilde{\Pi}_2^*(x)d\xi = \\ &= \alpha(x)\widetilde{P}_2(f(x)) + i\int_0^x f(\xi)\widetilde{P}_2\widehat{\Pi}_2(\xi)L\widehat{\Pi}_2^*(x)\widetilde{P}_2 d\xi, \end{aligned} \tag{7}$$

where $n \times m$ matrices $\widehat{\Pi}_1(\xi)$ and $\widehat{\Pi}_2(\xi)$ have the form

$$\widehat{\Pi}_1(\xi) = \begin{pmatrix} \widetilde{\Pi}_1(\xi) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} J_1, \quad \widehat{\Pi}_2(\xi) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \widetilde{\Pi}_2(\xi) \end{pmatrix} J_2.$$

The operator $\widetilde{T}_1$ is dissipative and the operator $\widetilde{T}_2$ is antidissipative on the subspaces $\widetilde{P}_1\mathbf{L}^2(0,l;\mathbb{C}^n)$ and $\widetilde{P}_2\mathbf{L}^2(0,l;\mathbb{C}^n)$ correspondingly.

Further instead of the colligations $\widetilde{X}_1$ and $\widetilde{X}_2$ we consider the colligations

$$\widehat{X}_1 = (\widetilde{P}_1\widetilde{T}_1\widetilde{P}_1; \widetilde{H}_1, \widehat{\Phi}_1, \mathbb{C}^m; L),$$

$$\widehat{X}_2 = (\widetilde{P}_2\widetilde{T}_2\widetilde{P}_2; \widetilde{H}_2, \widehat{\Phi}_2, \mathbb{C}^m; L),$$

where $\widetilde{H}_k = \widetilde{P}_k \mathbf{L}^2(0,l;\mathbb{C}^n)$,

$$\widehat{\Phi}_k f(x) = \int\limits_0^l f(x)\widehat{\Pi}_k(x)dx, \qquad k = 1,2.$$

Then the coupling $T = P_1TP_1 + P_2TP_2 + P_1TP_2$ of the operators T_1 и T_2 is unitary equivalent to the coupling of the operators $\widetilde{T}_1$ and $\widetilde{T}_2$ ($\widetilde{T}_1$ - a dissipative operator , $\widetilde{T}_2$ - an antidissipative operator) onto the principal subspaces. The unitary operator $U : \mathbf{L}^2(0,l;\mathbb{C}^n) \longrightarrow H$ defined by the equality

$$U = P_1U_1\widetilde{P}_1 + P_2U_2\widetilde{P}_2 \tag{8}$$

realizes this unitary equivalence.

But straightforward calculations show that

$$\begin{aligned} \widetilde{T} = U^*TU = \widetilde{P}_1U_1^*P_1TP_1U_1\widetilde{P}_1 + \widetilde{P}_2U_2^*P_2TP_2U_2\widetilde{P}_2+ \\ +\widetilde{P}_1U_1^*P_1TP_2U_2\widetilde{P}_2 = \\ = \widetilde{P}_1\widetilde{T}_1\widetilde{P}_1 + \widetilde{P}_2\widetilde{T}_2\widetilde{P}_2 + \widetilde{P}_1\widetilde{T}\widetilde{P}_2. \end{aligned} \tag{9}$$

where we have used the relations (5) and (8). On the other hand, using that $\widetilde{T}$ is a coupling, for the operator $\widetilde{P}_1\widetilde{T}\widetilde{P}_2$ we obtain that

$$\widetilde{P}_1\widetilde{T}\widetilde{P}_2 = i\widehat{\Phi}_1^*L\widehat{\Phi}_2\widetilde{P}_2f(x) = i\int\limits_0^l f(\xi)\widetilde{P}_2\widehat{\Pi}_2(\xi)L\widehat{\Pi}_1^*(x)\widetilde{P}_1d\xi. \tag{10}$$

Now the equalities (9), (6), (7) and (10) imply that

$$\begin{aligned} \widetilde{T}f(x) = \alpha(x)\widetilde{P}_1(f(x)) + i\int\limits_0^x f(\xi)\widetilde{P}_1\widehat{\Pi}_1(\xi)L\widehat{\Pi}_1^*(x)\widetilde{P}_1d\xi+ \\ +\alpha(x)\widetilde{P}_2(f(x)) + i\int\limits_0^x f(\xi)\widetilde{P}_2\widehat{\Pi}_2(\xi)L\widehat{\Pi}_2^*(x)\widetilde{P}_2d\xi+ \\ +i\int\limits_0^l f(\xi)\widetilde{P}_2\widehat{\Pi}_2(\xi)L\widehat{\Pi}_1^*(x)\widetilde{P}_1d\xi. \end{aligned} \tag{11}$$

Let us denote the next matrix

$$\Pi(x) = \begin{pmatrix} \widetilde{\Pi}_1(x) & 0 & 0 & 0 \\ 0 & 0 & 0 & \widetilde{\Pi}_2(x) \end{pmatrix}$$

Hence

$$\widetilde{P}_1\Pi(x)J_1 = \widehat{\Pi}_1(x), \qquad \widetilde{P}_2\Pi(x)J_2 = \widehat{\Pi}_2(x)$$

and the representation (11) takes the form

$$\begin{gathered}\widetilde{T}f(x) = \alpha(x)\widetilde{P}_1(f(x)) + i\int\limits_0^x f(\xi)\Pi(\xi)J_1LJ_1\Pi^*(x)d\xi + \\ +\alpha(x)\widetilde{P}_2(f(x)) + i\int\limits_0^x f(\xi)\Pi(\xi)J_2LJ_2\Pi^*(x)d\xi + \\ +i\int\limits_0^l f(\xi)\Pi(\xi)J_2LJ_1\Pi^*(x)d\xi\end{gathered}$$

Hence

$$\begin{gathered}\widetilde{T}f(x) = \alpha(x)f(x) + i\int\limits_0^x f(\xi)\Pi(\xi)J_1\Pi^*(x)d\xi - \\ -i\int\limits_0^x f(\xi)\Pi(\xi)J_2\Pi^*(x)d\xi + i\int\limits_0^l f(\xi)\Pi(\xi)S\Pi^*(x)d\xi.\end{gathered} \tag{12}$$

Consequently, from (12) it follows that the operator $T = P_1TP_1 + P_2TP_2 + P_1TP_2$ is unitary equivalent to the operator (12) (onto the principal subspace) and after direct calculations $\widetilde{T}$ takes the form

$$\begin{gathered}\widetilde{T}f(x) = \alpha(x)f(x) + i\int\limits_x^l f(\xi)\Pi(\xi)S\Pi^*(x)d\xi - \\ -i\int\limits_0^x f(\xi)\Pi(\xi)S^*\Pi^*(x)d\xi + i\int\limits_0^x f(\xi)\Pi(\xi)L\Pi^*(x)d\xi.\end{gathered}$$

The condition $\Pi^*(x)\Pi(x)J_1 = J_1\Pi^*(x)\Pi(x)$ is obvious.

The pfroof is complete. □

The proof of Theorem 1 together with the triangular model (1), introduced in [1], finishes the description of a large class of nonselfadjoint bounded nondissipative operators which are presented as a coupling of dissipative and antidissipative operators with finite dimensional imaginary parts and with real spectra, determined by a real nondecreasing function. This class of operators generates nondissipative continuous curves whose asymtotics exist and are obtained explicitly in the papers [7], [10], [8] in terms of multiplicative integrals and a finite dimensional analogue of the classical gamma-function. The explicit form of these asymptotics plays an important role in the construction of the scattering theory for the couple of operators (A^*, A) ([7], [10]) and results concerning the connection between the soliton theory and the theory of commuting nonselfadjoint operators ([4], [3]). A triangular model of regular couplings of dissipative and antidissipative operators for unbounded operators A with different domains of A and its adjoint A^* is introdused and investigated in [9, 10, 8].

REFERENCES:

1. G. S. Borisova, A new form of the triangular model of M.S. Livšic for a class of nondissipative operators, Comptes Rendus de l'Acadèmie bulgare des Sciences 53 (10) (2000) 9-12.
2. Galina S. Borisova, The operators $A_\gamma = \gamma A + \overline{\gamma} A^*$ for a class of nondissipative operators A with a limit of the corresponding correlation function, Serdica Math. J. 29 (2003) 109-140.
3. Galina S. Borisova, The connection between the Sturm-Liouville systems and the triangular model of couplings of dissipative and antidissipative operators, Comptes Rendus de l'Acadèmie bulgare des Sciences 69 (5) (2016) 563-572.
4. G. S. Borisova, K. P. Kirchev, Solitonic combinations and commuting nonselfadjoint operators, *Journal of Mathematical Analysis and Applications*, Volume 424, Issue 1, 2015, 21-48.
5. M. S. Brodskii, Triangular and Jordan representations of linear operators (Vol. 32). Transl. Math. Monographs, Amer. Math. Soc., 1971.
6. M. S. Brodskii, M. S. Livšic, Spectral analysis of non-self-adjoint operators and intermediate systems, Transl. Amer. Math. Soc. 13 (2) (1960) 265-346.
7. K. P. Kirchev, G. S. Borisova, Nondissipative curves in Hilbert spaces having a limit of the corresponding correlation function, Integral Equations Operator Theory 40 (2001) 309-341.
8. K. P. Kirchev, G. S. Borisova, Triangular models and asymptotics of continuous curves with bounded and unbounded semigroup generators, Serdica Math. J. 31 (2005) 95-174.
9. K. Kirchev, G. Borisova, A triangular model of regular couplings of dissipative and anridissipative operators, Comptes rendus de l'Acadèmie bulgare des sciences 58 (5) (2005) 481-486.
10. K. P. Kirchev, G. S. Borisova, Regular Couplings of Dissipative and Anti-Dissipative Unbounded Operators, Asymptotics of the Corresponding Non-Dissipative Processes and the Scattering Theory, Integral Equations Operator Theory 57 (2007) 339-379.

11. M.S. Livšic, Об одном классе линейных операторов в гильбертовом пространстве, *Математический сборник*, 1946, 19/61/2, 236-260.

12. M.S. Livšic, О спектральном разложении линейных операторов в гильбертовом пространстве, *Математический сборник*, 1954, 34/76/1, 145-198.

13. M.S. Livšic, On a certain class of linear operators in Hilbert space, *Mat. Sb.*, 19, (61), 1946, 236-260 (Russian).

14. M.S.Livšic, *Operators, oscillations, waves* (Open Systems). Transl. Math. Monographs 34, 1972.

15. M. S. Livšic, N. Kravitsky, A.S. Markus, V. Vinnikov, Theory of commuting nonselfadjoint operators, Kluwer Academic Publisher Group, Dordrecht, 1995.

16. M.S. Livšhic, A.A. Jancevich, Theory of Operator colligations in Hilbert space, Engl. Transl., J. Wiley, NY, 1979.

17. V.P. Potapov, The multiplicative structure of J-contractive matrix function, *Trudy Mosk. Mat. Obshch.*, 4, 1955, 125-236 (Russian).

Faculty of Mathematics and Informatics
Shumen University
9712 Shumen, Bulgaria
e-mail: g.borisova@shu.bg